

Quantum gravity corrections to the conformally coupled scalar self-mass-squared on de Sitter background

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We evaluate one loop quantum gravity corrections to the conformally coupled (CC) scalar self-mass-squared on a locally de Sitter background. In this paper we consider only the conformal-conformal interaction part of the self-mass-squared. This complements the minimal-minimal part worked out in the previous paper [15] and we will add the minimal-conformal part in a follow-up paper to complete the full self-mass-squared at one loop order. The computation is performed using dimensional regularization and the results are fully renormalized by absorbing divergences with counterterms. The finite results can give rise to quantum corrections to the CC scalar mode functions and therefore to their power spectra.

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I. INTRODUCTION

An intuitive way of understanding quantum effects is to examine the classical response to virtual particles. If there are not many virtual particles or they interact with the particles in question only weakly there will not be much quantum loop effect. In this respect it will be interesting to investigate the cases in which the number of virtual particles increases and their interactions with other particles. It has been known for a long time that the expansion of spacetime can lead to particle creation by delaying the annihilation of virtual pairs ripped out of the vacuum since Schrödinger first realized it [1]. In late 1960's Parker carried out explicit computations regarding particle production in expanding universe [2] and the results are summarized as follows: the effect is the strongest if the expansion is accelerated, and the virtual particles are massless and do not possess conformal symmetry [3].

The locally de Sitter background we consider is the most highly accelerated expansion with classical stability. The unique examples of massless particles with no conformal invariance are the massless, minimally coupled (MMC) scalars and gravitons. These particles are the sources of the scalar and tensor perturbations predicted by inflationary theories [4]. The scalar component of which has been mapped with great precision [5, 6]. The tensor components have not been mapped yet, but with the recent announcement for the detection of the B -mode polarization by the BICEP2 collaboration [7], it is not hopeless to image them in the near future.

The approximate tree order results for the scalar and tensor power spectra are

$$\Delta_{\mathcal{R}}^2(k) = \lim_{t \gg t_k} \frac{k^3}{2\pi^2} \times 4\pi G \times |v(t, k)|^2 \simeq \frac{GH^2}{\pi\epsilon}, \quad \Delta_h^2(k) = \lim_{t \gg t_k} \frac{k^3}{2\pi^2} \times 64\pi G \times |u(t, k)|^2 \simeq \frac{16GH^2}{\pi}. \quad (1)$$

Here $v(t, k)$ and $u(t, k)$ are the mode functions of scalar and tensor perturbations, G is Newton's constant, $H(t)$ is the Hubble parameter and $\epsilon(t)$ is the slow roll parameter. The time t is taken to be much later than t_k , the time of first horizon crossing for the mode of wave number k . One can study loop corrections to this lowest order effect using a mathematical object called the one-particle-irreducible (1PI) 2-point function. The procedure is first to compute the 1PI 2-point function, for the case of scalar it is the self-mass squared, $-iM^2(x; x')$. Second, to use it to quantum-correct the linearized effective field equation, for the CC scalar it is

$$\partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \phi(x) \right) - \frac{1}{6} R \phi(x) - \int d^4 x' M^2(x; x') \phi(x') = 0. \quad (2)$$

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By solving this equation one can get loop corrections to the mode functions for the inflaton, where the conformal coupling term can be thought effectively as an inflaton potential (similar to the $m^2\phi^2$ model) in the context of scalar-driven inflation models at the lowest order.

A number of computations in this regard have been made over the last decade [8–24, 27–38]. (A simple worked-out example specifically for loop corrections to the power spectra can be found in [39].) In this paper we carry out the first part of this procedure, that is we compute the self-mass-squared of CC scalar interacting with inflationary gravitons at one loop order and renormalize it so to make the integral in (2) finite. In a follow-up paper we will solve the effective field equation to study one loop corrections to the tree order mode functions of the CC scalar field.

One might expect strong quantum effects for the interaction between the MMC scalars and gravitons because these two particles (being massless and without conformal invariance) are the ones produced enormously during inflation [3]. However, many studies [8–20] suggest that the strongest effects are caused by non-derivative interactions. The reason why it happens is the differentiated scalar, so its kinetic energy is redshifted away during inflation so they interact very weakly with virtual gravitons. So one loop corrections from gravitons to the self-mass-squared of MMC scalar result in zero [15, 16]. The same phenomenon happens to the mirror case, namely the differentiated gravitons interacting with virtual scalars [17, 20]. In fact, these null results motivated the authors to consider the conformally coupled scalar interacting with virtual gravitons by noting that in the conformal coupling term the scalar is not differentiated. Moreover, the mode functions of conformally coupled scalar redshift to zero at tree order. Therefore any loop corrections to them would be dominant over the zero tree order result.

In Section II, we derive a formal expression for the CC scalar self-mass-squared. At one-loop order it consists of two parts, namely 4-point and 3-point interactions. Explicit computations of the two parts are performed in Section III. The results are fully renormalized using appropriate counterterms in Section IV. Our discussions comprise Section V.

II. THE SELF-MASS-SQUARED

The Lagrangian which describes pure gravity and the interaction between gravitons and the conformally coupled scalar (in D spacetime dimensions to facilitate dimensional regularization) is

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial_\nu\phi g^{\mu\nu}\sqrt{-g} - \frac{D-2}{8(D-1)}\phi^2 R\sqrt{-g} + \frac{1}{16\pi G}(R - (D-2)\Lambda)\sqrt{-g}, \quad (3)$$

where R is the Ricci scalar and $\Lambda = (D-1)H^2$ is the cosmological constant. Varying the 1PI effective action corresponding to the Lagrangian (3) with respect to the scalar field we have the linearized effective field equation (2),

$$\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi(x)\right) - \frac{1}{6}R\phi(x) - \int d^4x' M^2(x; x')\phi(x') = 0.$$

We work on the open conformal coordinate patch of de Sitter space

$$ds^2 = \hat{g}_{\mu\nu}dx^\mu dx^\nu = a^2(\eta)\left[-d\eta^2 + d\vec{x}\cdot d\vec{x}\right], \quad \text{where} \quad a(\eta) = -\frac{1}{H\eta}, \quad (4)$$

with the coordinate ranges

$$-\infty < x^0 \equiv \eta < 0, \quad -\infty < x^i < +\infty, \quad i = 1, 2, \dots, D-1. \quad (5)$$

Perturbation theory is expressed in terms of the background metric $\hat{g}_{\mu\nu}$ and conformally rescaled graviton field $h_{\mu\nu}$

$$g_{\mu\nu}(x) \equiv a^2\left[\eta_{\mu\nu} + \kappa h_{\mu\nu}(x)\right], \quad \text{where} \quad \kappa^2 \equiv 16\pi G, \quad (6)$$

Due to the conformal coupling term (the second term) the matter sector (the first and second term) in the Lagrangian (3) is invariant under the conformal rescaling,

$$g_{\mu\nu} \equiv \Omega^2 \tilde{g}_{\mu\nu}, \quad \phi \equiv \Omega^{\frac{2-D}{2}} \tilde{\phi}. \quad (7)$$

$$\Rightarrow \mathcal{L}_{\text{Mat}} = -\frac{1}{2}\partial_\mu\phi\partial_\nu\phi g^{\mu\nu}\sqrt{-g} - \frac{D-2}{8(D-1)}\phi^2 R\sqrt{-g} = -\frac{1}{2}\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi}\tilde{g}^{\mu\nu}\sqrt{-\tilde{g}} - \frac{D-2}{8(D-1)}\tilde{\phi}^2 \tilde{R}\sqrt{-\tilde{g}}. \quad (8)$$

Taking $\Omega = a$ we can simply work with the conformally rescaled metric

$$\tilde{g}_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}. \quad (9)$$

of which the inverse and the volume element are expanded as

$$\tilde{g}^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^\mu{}_\rho h^{\rho\nu} + \mathcal{O}(\kappa^3), \quad (10)$$

$$\sqrt{-\tilde{g}} = 1 + \frac{1}{2}\kappa h + \frac{1}{8}\kappa^2 h^2 - \frac{1}{4}\kappa^2 h^{\rho\sigma} h_{\rho\sigma} + \mathcal{O}(\kappa^3). \quad (11)$$

The next step is using the above expressions for the metric in order to expand the Ricci scalar at second order in κ . The expression for \tilde{R} is,

$$\begin{aligned} \tilde{R} = \kappa \left(-h^\mu{}_\mu + h^{\mu\nu}{}_{,\mu\nu} \right) + \kappa^2 \left(-2h^{\mu\nu} h_{\lambda\nu,\mu}{}^\lambda + h^{\mu\nu} h_{\mu\nu,\lambda}{}^\lambda + h^{\mu\nu} h_{,\mu\nu} + \frac{3}{4} h^{\mu\nu}{}_{,\lambda} h_{\mu\nu}{}^\lambda + h^{\mu\nu}{}_{,\mu} h_{,\nu} - h^{\mu\nu}{}_{,\mu} h_{\lambda\nu}{}^\lambda \right. \\ \left. - \frac{1}{2} h^{\mu\nu}{}_{,\lambda} h^\lambda{}_{\nu,\mu} - \frac{1}{4} h_{,\mu} h^\mu{}_{,\nu} - \frac{1}{2} h h^\mu{}_\mu + \frac{1}{2} h h^{\mu\nu}{}_{,\mu\nu} \right) + \mathcal{O}(\kappa^3). \end{aligned} \quad (12)$$

Using the perturbative expansion, the self-mass-squared $-iM^2(x; x')$ can be computed at any desired loop order. Our aim is to derive it at one loop order, which consists of the three Feynman diagrams depicted in Figure 1. The first

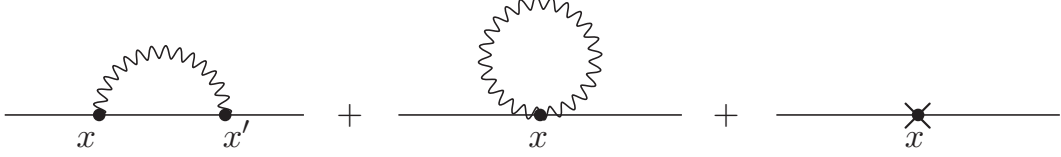


FIG. 1: The one-loop self-mass-squared from massless conformally coupled scalar. The first two diagrams represent the 3-point and 4-point interactions, respectively. The last diagram corresponds to the counterterms required to absorb ultraviolet divergences from the two primitive diagrams.

two diagrams which represent the 3-point and 4-point interactions, respectively have the following analytic expressions.

$$\text{1st diagram: } -iM_{3\text{pt}}^2(x; x') \equiv \left\langle i \frac{\delta S[\phi, h]}{\delta \phi(x)} i \frac{\delta S[\phi, h]}{\delta \phi(x')} \right\rangle_0, \quad (13)$$

$$\text{2nd diagram: } -iM_{4\text{pt}}^2(x; x') \equiv \left\langle i \frac{\delta^2 S[\phi, h]}{\delta \phi(x) \delta \phi(x')} \right\rangle_0, \quad (14)$$

where the subscript 0 on the expectation value indicates that it is to be taken in the free theory. Once the action $S = \int d^D x \mathcal{L}$ is expanded computing these two expressions is straightforward. The 3-point and 4-point diagrams correspond to the $\tilde{\phi}^2 h$ and $\tilde{\phi}^2 h^2$ interactions which derive from expanding the matter part of the Lagrangian (8). The expansion of the scalar kinetic term is given in the previous work [15] and therefore we focus on the conformal coupling term in (8).

$$\begin{aligned} \mathcal{L}_{\text{CC}} &\equiv -\frac{D-2}{8(D-1)} \tilde{\phi}^2 \tilde{R} \sqrt{-\tilde{g}} \\ &= -\frac{D-2}{8(D-1)} \kappa \tilde{\phi}^2 \left(-h^\mu{}_\mu + h^{\mu\nu}{}_{,\mu\nu} \right) - \frac{D-2}{8(D-1)} \kappa^2 \tilde{\phi}^2 \left(-2h^{\mu\nu} h_{\lambda\nu,\mu}{}^\lambda + h^{\mu\nu} h_{\mu\nu,\lambda}{}^\lambda + h^{\mu\nu} h_{,\mu\nu} \right. \\ &\quad \left. + \frac{3}{4} h^{\mu\nu}{}_{,\lambda} h_{\mu\nu}{}^\lambda + h^{\mu\nu}{}_{,\mu} h_{,\nu} - h^{\mu\nu}{}_{,\mu} h_{\lambda\nu}{}^\lambda - \frac{1}{2} h^{\mu\nu}{}_{,\lambda} h^\lambda{}_{\nu,\mu} - \frac{1}{4} h_{,\mu} h^\mu{}_{,\nu} - \frac{1}{2} h h^\mu{}_\mu + \frac{1}{2} h h^{\mu\nu}{}_{,\mu\nu} \right). \end{aligned} \quad (15)$$

In the following subsection we derive the formal expressions for the self-mass-squared from the 3-point and 4-point interactions in the conformal coupling term. For notational simplicity we drop $\tilde{}$ from now on, but remember that our metric, the scalar field ϕ and graviton field $h_{\mu\nu}$ are all conformally rescaled ones:

$$\tilde{g}_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \quad \tilde{\phi} \equiv \phi. \quad (16)$$

A. Formal expressions for the one loop self-mass-squared

1. 4-point contributions

The 4-point contributions to the self-mass-squared come from $\phi^2 h^2$ terms in (15). Let us illustrate the derivation by calculating it from the first term,

$$\mathcal{L}_{4a} \equiv -\frac{D-2}{8(D-1)} \kappa^2 \phi^2 (-2h^{\mu\nu} h_{\lambda\nu,\mu}{}^\lambda). \quad (17)$$

From the defining expression (14), the first step is to vary the action with respect to the scalar field,

$$\begin{aligned} \frac{\delta^2 S_{4a}}{\delta\phi(x')\delta\phi(x)} &= -\frac{D-2}{8(D-1)} \frac{\delta}{\delta\phi(x')} \int d^D y \left[-2\kappa^2 \frac{\delta\phi^2(y)}{\delta\phi(x)} h^{\mu\nu}(y) \partial^\lambda \partial_\mu h_{\mu\nu}(y) \right], \\ &= -\frac{D-2}{8(D-1)} \left[-4\kappa^2 \delta^D(x-x') h^{\mu\nu}(x) \partial^\lambda \partial_\mu h_{\mu\nu}(x) \right], \\ &= -\frac{D-2}{8(D-1)} \left[-4\kappa^2 \delta^D(x-x') h^{\mu\nu}(x') \partial^\lambda \partial_\mu h_{\mu\nu}(x) \right]. \end{aligned} \quad (18)$$

In the last line we used the delta function to convert $h^{\mu\nu}(x)$ to $h^{\mu\nu}(x')$ in order to distinguish them from one another. It also is useful to see the tensorial structure of the graviton propagator from x to x' . The self-mass-squared is the functional integral of i times (18) over the relevant fields,

$$\begin{aligned} -iM_{4a}^2(x; x') &\equiv \left\langle i \frac{\delta^2 S_{4a}}{\delta\phi(x')\delta\phi(x)} \right\rangle_0, \\ &= \frac{D-2}{2(D-1)} \kappa^2 i \delta^D(x-x') \partial^\lambda \partial_\mu \left\langle h^{\mu\nu}(x) h_{\lambda\nu}(x') \right\rangle_0, \\ &= \frac{32(D-1)}{(D-2)} \tilde{\kappa}^2 i \delta^D(x-x') \partial^\lambda \partial_\mu \left\{ i [\mu\nu \Delta_{\lambda\nu}](x; x') \right\}. \end{aligned} \quad (19)$$

Here we define $\tilde{\kappa} \equiv \frac{D-2}{8(D-1)} \times \kappa$ for future convenience. Also note that an expression for the graviton propagator emerges in the last step:

$$i[\mu\nu \Delta_{\lambda\nu}](x; x') = \left\langle h^{\mu\nu}(x) h_{\lambda\nu}(x') \right\rangle_0. \quad (20)$$

Using the same procedure for the remaining terms in (15) we have the contributions from the 4-point vertices as

$$\begin{aligned} -iM_{4\text{pt}}^2(x; x') &= \frac{32(D-1)}{(D-2)} \tilde{\kappa}^2 i \delta^D(x-x') \left\{ \partial^\lambda \partial'_\mu i[\mu\nu \Delta_{\lambda\nu}](x; x') - \frac{1}{2} \partial^\lambda \partial'_\lambda i[\mu\nu \Delta_{\mu\nu}](x; x') - \frac{1}{2} \partial_\mu \partial'_\nu i[\mu\nu \Delta^\alpha_\alpha](x; x') \right. \\ &\quad - \frac{3}{8} \partial^\lambda \partial'_\lambda i[\mu\nu \Delta_{\mu\nu}](x; x') - \frac{1}{2} \partial_\mu \partial'_\nu i[\alpha_\alpha \Delta^{\mu\nu}](x; x') + \frac{1}{2} \partial^\lambda \partial'_\mu i[\mu\nu \Delta_{\lambda\nu}](x; x') + \frac{1}{4} \partial^\mu \partial'_\lambda i[\lambda\nu \Delta_{\mu\nu}](x; x') \\ &\quad \left. + \frac{1}{8} \partial_\mu \partial'^\mu i[\alpha_\alpha \Delta^\beta_\beta](x; x') + \frac{1}{4} \partial^\lambda \partial'_\lambda i[\alpha_\alpha \Delta^\beta_\beta](x; x') - \frac{1}{4} \partial_\mu \partial'_\nu i[\alpha_\alpha \Delta^{\mu\nu}](x; x') \right\}. \end{aligned} \quad (21)$$

2. 3-point contributions

The 3-point contributions derive from $\phi^2 h$ terms in (15):

$$\mathcal{L}_{3\text{pt}} = -\frac{D-2}{8(D-1)} \kappa \phi^2 (-h^\mu_{\cdot\mu} + h^{\mu\nu}_{\cdot\mu\nu}) = -\tilde{\kappa} \phi^2 (-h^\mu_{\cdot\mu} + h^{\mu\nu}_{\cdot\mu\nu}) \equiv \mathcal{L}_{3a} + \mathcal{L}_{3b}. \quad (22)$$

The 3-point self-mass-squared defined in (13) then can be written as

$$-iM_{3\text{pt}}^2(x; x') = \left\langle i \frac{\delta S_{3a}}{\delta\phi(x)} i \frac{\delta S_{3a}}{\delta\phi(x')} + i \frac{\delta S_{3a}}{\delta\phi(x)} i \frac{\delta S_{3b}}{\delta\phi(x')} + i \frac{\delta S_{3b}}{\delta\phi(x)} i \frac{\delta S_{3a}}{\delta\phi(x')} + i \frac{\delta S_{3b}}{\delta\phi(x)} i \frac{\delta S_{3b}}{\delta\phi(x')} \right\rangle_0. \quad (23)$$

Here the variations of the action with respect to the scalar are

$$\frac{\delta S_{3a}}{\delta\phi(x)} = -\tilde{\kappa} \frac{\delta}{\delta\phi(x)} \int d^D y \left[-\phi^2(y) \partial_\mu \partial^\mu h(y) \right] = -\tilde{\kappa} \left[-2\phi(x) \partial_\mu \partial^\mu h(x) \right], \quad (24)$$

$$\frac{\delta S_{3b}}{\delta\phi(x)} = -\tilde{\kappa} \frac{\delta}{\delta\phi(x)} \int d^D y \left[\phi^2(y) \partial_\mu \partial_\nu h^{\mu\nu}(y) \right] = -\tilde{\kappa} \left[2\phi(x) \partial_\mu \partial_\nu h^{\mu\nu}(x) \right]. \quad (25)$$

By substituting (24), (25) into (23), we obtain

$$-iM_{3\text{pt}}^2(x; x') = -\tilde{\kappa}^2 i \Delta_{\text{cf}} \left\{ \partial^2 \partial'^2 i[\alpha_\alpha \Delta^\beta_\beta](x; x') - [\partial_\mu \partial_\nu \partial'^2 + \partial'_\mu \partial'_\nu \partial^2] i[\alpha_\alpha \Delta^{\mu\nu}](x; x') + \partial_\mu \partial_\nu \partial'_\alpha \partial'_\beta i[\mu\nu \Delta^{\alpha\beta}](x; x') \right\}. \quad (26)$$

Here note that the appearance of the propagator for a massless, conformally coupled scalar

$$i\Delta_{\text{cf}}(x; x') = \left\langle \phi(x)\phi(x') \right\rangle_0. \quad (27)$$

In fact, there exists another kind of 3-point contributions taking one variation from the kinetic term and the other from the conformal coupling in the Lagrangian (8). The 3-point interactions from the scalar kinetic term are

$$\mathcal{L}_{\text{K3pt}} = -\frac{1}{2}\kappa\partial_\mu\phi\partial_\nu\phi\left(-h^{\mu\nu} + \frac{1}{2}\eta^{\mu\nu}h\right) \equiv \mathcal{L}_{K3c} + \mathcal{L}_{K3d}, \quad (28)$$

and the 3-point self-mass-squared from the ‘cross terms’ can be computed as

$$-iM_{\text{3ptcross}}^2(x; x') = \left\langle i\frac{\delta S_{3a}}{\delta\phi(x)}i\frac{\delta S_{K3c}}{\delta\phi(x')} + i\frac{\delta S_{3a}}{\delta\phi(x)}i\frac{\delta S_{K3d}}{\delta\phi(x')} + i\frac{\delta S_{3b}}{\delta\phi(x)}i\frac{\delta S_{K3c}}{\delta\phi(x')} + i\frac{\delta S_{3b}}{\delta\phi(x)}i\frac{\delta S_{K3d}}{\delta\phi(x')} \right\rangle_0. \quad (29)$$

Including (26), (29) and the contributions from the kinetic terms given in [15] (which can be formally written as)

$$-iM_{\text{3ptK}}^2(x; x') = \left\langle i\frac{\delta S_{K3c}}{\delta\phi(x)}i\frac{\delta S_{K3c}}{\delta\phi(x')} + i\frac{\delta S_{K3c}}{\delta\phi(x)}i\frac{\delta S_{K3d}}{\delta\phi(x')} + i\frac{\delta S_{K3d}}{\delta\phi(x)}i\frac{\delta S_{K3c}}{\delta\phi(x')} + i\frac{\delta S_{K3d}}{\delta\phi(x)}i\frac{\delta S_{K3d}}{\delta\phi(x')} \right\rangle_0. \quad (30)$$

completes the 3-point contributions. However, with the experience from [15] and the present work, each of the three expressions (26), (29) and (30) takes a number of heavy computations and can be treated separately. Thus we leave calculating the cross contributions as a follow-up project.

B. Propagators

From the 4-point and 3-point contributions (21) and (26), we see that the propagators are the main ingredients of the the self-mass-squared. This subsection therefore provides with the scalar and graviton propagators. We follow three notational conventions employed in the previous work [15] for continuity and comparison with the current results. First, the background geometry is denoted with a hat, which was already introduced in (4),

$$\hat{g}_{\mu\nu} = a^2\eta_{\mu\nu} \quad \text{accordingly} \quad \hat{R} = D(D-1)H^2. \quad (31)$$

Second, considering the gauge choice for the graviton propagator in which time and space are treated differently, it is convenient to differentiate the spatial parts of the Lorentz metric and the Kronecker delta with a bar,

$$\bar{\eta}_{\mu\nu} \equiv \eta_{\mu\nu} + \delta_\mu^0\delta_\nu^0 \quad \text{and} \quad \bar{\delta}_\nu^\mu \equiv \delta_\nu^\mu - \delta_0^\mu\delta_\nu^0. \quad (32)$$

Third, it is useful to define the de Sitter length function $y(x; x')$ in terms of the de Sitter invariant length $\ell(x; x')$ from x^μ to x'^μ :

$$y(x; x') = 4\sin^2\left(\frac{1}{2}H\ell(x; x')\right) = aa'H^2\left\{\|\vec{x} - \vec{x}'\|^2 - \left(|\eta - \eta'| - i\delta\right)^2\right\}, \quad (33)$$

where $a \equiv a(\eta)$ and $a' \equiv a(\eta')$. The propagator for a massless conformally coupled scalar has long been known [40],

$$i\Delta_{\text{cf}}(x; x') = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\Gamma\left(\frac{D}{2}-1\right)\left(\frac{4}{y}\right)^{\frac{D}{2}-1}. \quad (34)$$

The graviton propagator was derived by adding the gauge fixing term to the Lagrangian [41],

$$\mathcal{L}_{\text{GF}} = -\frac{1}{2}a^{D-2}\eta^{\mu\nu}F_\mu F_\nu, \quad F_\mu \equiv \eta^{\rho\sigma}\left(h_{\mu\rho,\sigma} - \frac{1}{2}h_{\rho\sigma,\mu} + (D-2)Hah_{\mu\rho}\delta_\sigma^0\right). \quad (35)$$

The quadratic part of the gauge fixed Lagrangian can be partially integrated to extract the kinetic operator $D_{\mu\nu}^{\rho\sigma}$ as

$$\frac{1}{2}h^{\mu\nu}D_{\mu\nu}^{\rho\sigma}h_{\rho\sigma}, \quad (36)$$

where

$$D_{\mu\nu}{}^{\rho\sigma} \equiv \left\{ \frac{1}{2} \bar{\delta}_\mu^{(\rho} \bar{\delta}_\nu^{\sigma)} - \frac{1}{4} \eta_{\mu\nu} \eta^{\rho\sigma} - \frac{1}{2(D-3)} \delta_\mu^0 \delta_\nu^0 \delta_0^\rho \delta_0^\sigma \right\} D_A + \delta_{(\mu}^0 \bar{\delta}_{\nu)}^{(\rho} \delta_0^{\sigma)} D_B + \frac{1}{2} \left(\frac{D-2}{D-3} \right) \delta_\mu^0 \delta_\nu^0 \delta_0^\rho \delta_0^\sigma D_C . \quad (37)$$

The three scalar differential operators are defined as

$$D_A \equiv \partial_\mu \left(\sqrt{-\hat{g}} \hat{g}^{\mu\nu} \partial_\nu \right) , \quad (38)$$

$$D_B \equiv \partial_\mu \left(\sqrt{-\hat{g}} \hat{g}^{\mu\nu} \partial_\nu \right) - \frac{1}{D} \left(\frac{D-2}{D-1} \right) \hat{R} \sqrt{-\hat{g}} , \quad (39)$$

$$D_C \equiv \partial_\mu \left(\sqrt{-\hat{g}} \hat{g}^{\mu\nu} \partial_\nu \right) - \frac{2}{D} \left(\frac{D-3}{D-1} \right) \hat{R} \sqrt{-\hat{g}} . \quad (40)$$

The graviton propagator should obey the following defining equation,

$$D_{\mu\nu}{}^{\rho\sigma} \times i \left[{}_{\rho\sigma} \Delta^{\alpha\beta} \right] (x; x') = \delta_\mu^{(\alpha} \delta_\nu^{\beta)} i \delta^D(x - x') , \quad (41)$$

and by inverting the kinetic operator one can get the graviton propagator. This inversion procedure can be done for each scalar kinetic operator by setting the graviton propagator as a sum of constant tensor factors times scalar propagators:

$$i[{}_{\mu\nu} \Delta_{\rho\sigma}] (x; x') = \sum_{I=A,B,C} [{}_{\mu\nu} T_{\rho\sigma}^I] i \Delta_I(x; x') , \quad (42)$$

where the scalar propagators satisfy

$$D_I \times i \Delta_I(x; x') = i \delta^D(x - x') \quad \text{for} \quad I = A, B, C . \quad (43)$$

Here the tensor factors are given as

$$[{}_{\mu\nu} T_{\rho\sigma}^A] = 2 \bar{\eta}_{\mu(\rho} \bar{\eta}_{\sigma)\nu} - \frac{2}{D-3} \bar{\eta}_{\mu\nu} \bar{\eta}_{\rho\sigma} , \quad (44)$$

$$[{}_{\mu\nu} T_{\rho\sigma}^B] = -4 \delta_{(\mu}^0 \bar{\eta}_{\nu)(\rho} \delta_{\sigma)}^0 , \quad (45)$$

$$[{}_{\mu\nu} T_{\rho\sigma}^C] = \frac{2}{(D-3)(D-2)} [(D-3) \delta_\mu^0 \delta_\nu^0 + \bar{\eta}_{\mu\nu}] [(D-3) \delta_\rho^0 \delta_\sigma^0 + \bar{\eta}_{\rho\sigma}] . \quad (46)$$

The A-type propagator which is the same as the one for the MMC scalar has de Sitter invariant (as a function of y) and breaking parts [42, 43].

$$i \Delta_A(x; x') = A(y) + k \ln(aa') , \quad (47)$$

where $k \equiv \frac{H^{D-2}}{(4\pi)^{D/2}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}$. The de Sitter invariant part $A(y)$ is [43],

$$A(y) \equiv \frac{H^{D-2}}{(4\pi)^{D/2}} \left\{ \Gamma\left(\frac{D}{2} - 1\right) \left(\frac{4}{y}\right)^{\frac{D}{2}-1} + \frac{\Gamma(\frac{D}{2} + 1)}{\frac{D}{2} - 2} \left(\frac{4}{y}\right)^{\frac{D}{2}-2} - \pi \cot\left(\frac{\pi D}{2}\right) \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \right. \\ \left. + \sum_{n=1}^{\infty} \left[\frac{1}{n} \frac{\Gamma(n+D-1)}{\Gamma(n+\frac{D}{2})} \left(\frac{y}{4}\right)^n - \frac{1}{n-\frac{D}{2}+2} \frac{\Gamma(n+\frac{D}{2}+1)}{\Gamma(n+2)} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} \right] \right\} . \quad (48)$$

Note that this de Sitter breaking solution still preserves homogeneity and isotropy and it is a well-known issue that there is no de Sitter invariant solution for the MMC scalar propagator [44].

The B-type and C-type propagators have the following de Sitter invariant solutions

$$i \Delta_B(x; x') = i \Delta_{\text{cf}}(x; x') - \frac{H^{D-2}}{(4\pi)^{D/2}} \sum_{n=0}^{\infty} \left\{ \frac{\Gamma(n+D-2)}{\Gamma(n+\frac{D}{2})} \left(\frac{y}{4}\right)^n - \frac{\Gamma(n+\frac{D}{2})}{\Gamma(n+2)} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} \right\} , \quad (49)$$

$$i \Delta_C(x; x') = i \Delta_{\text{cf}}(x; x') + \frac{H^{D-2}}{(4\pi)^{D/2}} \sum_{n=0}^{\infty} \left\{ (n+1) \frac{\Gamma(n+D-3)}{\Gamma(n+\frac{D}{2})} \left(\frac{y}{4}\right)^n - \left(n-\frac{D}{2}+3\right) \frac{\Gamma(n+\frac{D}{2}-1)}{\Gamma(n+2)} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} \right\} . \quad (50)$$

The infinite series terms of a positive power of y vanish for $D = 4$ so that one only need to retain them when multiplying a fixed divergence. This makes these propagators and loop calculations manageable.

III. COMPUTATION OF THE ONE LOOP SELF-MASS-SQUARED

In this section we evaluate the formal expressions for the 4-point and 3-point contributions to the self-mass-squared given in (21) and (26). The contribution from the 4-point vertices turns out to be finite. To manage the divergences occurring in the 3-point interactions we put them in the form of external operators acting on functions of y . This form is convenient for renormalization in the next section.

A. Contribution from the 4-point vertices

We start with the analytic expression for the 4-point contribution $-iM_{4\text{pt}}^2(x; x')$ in (21) corresponding to the following Feynman diagram, Fig. 2.

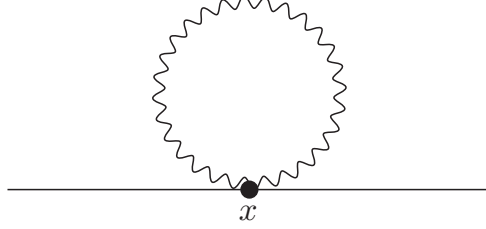


FIG. 2: Contribution from the 4-point vertices.

We take the coincidence limit with the aid of the delta function. Again we work it out for the first term to demonstrate the procedure,

$$\delta^D(x - x') \partial^\lambda \partial_\mu i[\mu^\nu \Delta_{\lambda\nu}](x; x') = \lim_{x \rightarrow x'} \partial^\lambda \partial_\mu i[\mu^\nu \Delta_{\lambda\nu}](x; x'). \quad (51)$$

The constant tensor factors of this term are evaluated from (44), (45) and (46),

$$[\mu^\nu T_{\lambda\nu}^A] = \frac{D^2 - 3D - 2}{D - 3} \bar{\delta}_\lambda^\mu, \quad (52)$$

$$[\mu^\nu T_{\lambda\nu}^B] = \bar{\delta}_\lambda^\mu + (D - 1) \delta_0^\mu \delta_\lambda^0, \quad (53)$$

$$[\mu^\nu T_{\lambda\nu}^C] = \frac{2}{(D - 3)(D - 2)} [(D - 3)^2 \delta_0^\mu \delta_\lambda^0 + \bar{\delta}_\lambda^\mu]. \quad (54)$$

and the differentiated scalar propagators become

$$\lim_{x \rightarrow x'} \partial^\lambda \partial_\mu i[\Delta_A](x; x') = \frac{H^D a^2}{(4\pi)^{D/2}} \frac{\Gamma(D - 1)}{\Gamma(\frac{D}{2})} \left\{ \frac{D - 1}{D} \delta_\mu^\lambda - \delta_0^\lambda \delta_\mu^0 \right\}, \quad (55)$$

$$\lim_{x \rightarrow x'} \partial^\lambda \partial_\mu i[\Delta_B](x; x') = \frac{H^D a^2}{(4\pi)^{D/2}} \frac{\Gamma(D - 1)}{\Gamma(\frac{D}{2})} \left\{ -\frac{1}{D} \delta_\mu^\lambda \right\}, \quad (56)$$

$$\lim_{x \rightarrow x'} \partial^\lambda \partial_\mu i[\Delta_C](x; x') = \frac{H^D a^2}{(4\pi)^{D/2}} \frac{\Gamma(D - 1)}{\Gamma(\frac{D}{2})} \left\{ \frac{2}{D(D - 2)} \delta_\mu^\lambda \right\}. \quad (57)$$

Therefore for the first term we have,

$$\begin{aligned} \lim_{x \rightarrow x'} \partial^\lambda \partial_\mu [\mu^\nu \Delta_{\lambda\nu}](x; x') &= \frac{H^D a^2}{(4\pi)^{D/2}} \frac{\Gamma(D - 1)}{\Gamma(\frac{D}{2})} \frac{(D^4 - 6D^3 + 9D^2 + 4D - 16)}{(D - 2)^2}, \\ &\rightarrow \frac{H^4 a^2}{16\pi^2} \times 8 \quad \text{in } D = 4. \end{aligned} \quad (58)$$

Using the same procedure for the remaining nine terms we obtain the 4-point contributions in $D = 4$ dimensions,

$$\begin{aligned}
-iM_{4\text{pt}}^2(x; x') &= \frac{i\tilde{\kappa}^2}{3}\delta^4(x-x')\frac{H^4a^2}{16\pi^2} \times \left\{ 1 \times (8) - \frac{1}{2} \times (20) - \frac{1}{2} \times (-16) - \frac{3}{8} \times (-32) \right. \\
&\quad \left. - \frac{1}{2} \times (16) + \frac{1}{2} \times (-8) + \frac{1}{4} \times (-8) + \frac{1}{8} \times (64) + \frac{1}{4} \times (-40) - \frac{1}{4} \times (-16) \right\}, \\
&= 2i\kappa^2 \times \frac{H^4a^2}{16\pi^2} \delta^4(x-x').
\end{aligned} \tag{59}$$

B. 3-point interaction

In this subsection we evaluate the 3-point contribution $-iM_{3\text{pt}}^2(x; x')$ in (26) depicted in the following diagram, Fig. 3.

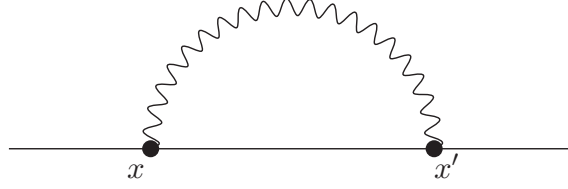


FIG. 3: Contribution from the 3-point vertices.

Substituting the graviton propagator and contracting the tensor indices in (26) we have

$$\begin{aligned}
&= -\tilde{\kappa}^2 i\Delta_{\text{cf}} \left\{ -\frac{4(D-1)}{D-3} \partial^2 \partial'^2 i\Delta_A + \frac{8}{(D-3)(D-2)} \partial^2 \partial'^2 i\Delta_C - \frac{4}{D-3} \left[-\nabla^2 \partial'^2 i\Delta_A \right. \right. \\
&\quad \left. -\nabla^2 \partial^2 i\Delta_A + \frac{1}{D-2} \left[(D-3) \partial_0^2 + \nabla^2 \right] \partial'^2 i\Delta_C + \frac{1}{D-2} \left[(D-3) \partial_0'^2 + \nabla^2 \right] \partial^2 i\Delta_C \right] + \frac{2(D-4)}{D-3} \nabla^4 i\Delta_A \\
&\quad \left. + 4\nabla^2 \partial_0 \partial_0' i\Delta_B + \frac{2}{(D-3)(D-2)} \left[(D-3)^2 \partial_0^2 \partial_0'^2 + (D-3) \nabla^2 (\partial_0'^2 + \partial_0^2) + \nabla^4 \right] i\Delta_C \right\}.
\end{aligned} \tag{60}$$

Here we applied

$$\partial'_i = -\partial_i, \quad \nabla'^2 = \nabla^2, \quad \partial^2 = -\partial_0^2 + \nabla^2. \tag{61}$$

Also, in the de Sitter background the covariant d'Alembertian operator is expressed as

$$\square \equiv \frac{1}{\sqrt{-\hat{g}}} \partial_\mu \left(\sqrt{-\hat{g}} \hat{g}^{\mu\nu} \partial_\nu \right) = -\frac{1}{a^2} \partial_0^2 - \frac{(D-2)H}{a} \partial_0 + \frac{1}{a^2} \nabla^2. \tag{62}$$

Let us remember that the expression for the conformal propagator was defined for de Sitter geometry in (34). Since we conformally rescaled our metric to flat space and the CC scalar is invariant under this rescaling we can work with the flat space limit of $i\Delta_{\text{cf}}$. In order to do the calculations more systematically we express all the propagators in terms of the de Sitter invariant function $y(x; x')$ in (33),

$$\begin{aligned}
i\Delta_{\text{cf}} &= \frac{\Gamma\left(\frac{D-1}{2}\right)}{(4\pi)^{\frac{D}{2}}} \left(\frac{4}{\Delta x^2}\right)^{\frac{D}{2}-1} = (aa')^{\frac{D}{2}-1} \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \Gamma\left(\frac{D-1}{2}\right) \left(\frac{4}{y}\right)^{\frac{D}{2}-1} \equiv (aa')^{\frac{D}{2}-1} F(y), \\
i\Delta_{A,B,C} &\equiv A(y), B(y), C(y).
\end{aligned} \tag{63}$$

Obviously a similar flat space version for the graviton can not be used for this calculation since the graviton is not conformally invariant. Note also that the de Sitter breaking term in $i\Delta_A$ is dropped. Then the terms in the 3-point contribution can be classified as the following four types,

$$F(y) \partial_0^2 \partial_0'^2 A(y), \quad F(y) \nabla^4 A(y), \quad F(y) \nabla^2 [\partial_0^2 + \partial_0'^2] A(y) \quad \text{and} \quad F(y) \nabla^2 \partial_0 \partial_0' A(y). \tag{65}$$

This allows to write the 3-point contribution as

$$-iM_{3\text{pt}}^2(x; x') = -\tilde{\kappa}^2(aa')^{\frac{D}{2}-1} \left\{ C_{1a}F(y)\partial_0^2\partial_0'^2A(y) + C_{2a}F(y)\nabla^4A(y) + C_{3a}F(y)\nabla^2[\partial_0^2 + \partial_0'^2]A(y) \right. \\ \left. + C_bF(y)\nabla^2\partial_0\partial_0'B(y) + C_{1c}F(y)\partial_0^2\partial_0'^2C(y) + C_{2c}F(y)\nabla^4C(y) + C_{3c}F(y)\nabla^2[\partial_0^2 + \partial_0'^2]C(y) \right\}. \quad (66)$$

Here the coefficients are

$$C_{1a} \equiv -4\left(\frac{D-1}{D-3}\right), \quad C_{2a} \equiv -2\left(\frac{D-2}{D-3}\right), \quad C_{3a} \equiv 4\left(\frac{D-2}{D-3}\right), \quad C_b \equiv 4, \\ C_{1c} \equiv \frac{2(D-2)^2}{(D-3)(D-2)}, \quad C_{2c} \equiv \frac{2}{(D-3)(D-2)} \quad \text{and} \quad C_{3c} \equiv \frac{(D-1)}{(D-3)(D-2)}. \quad (67)$$

Recalling the self-mass-squared is eventually integrated over 4-dimensional spacetime in the effective field equation (2), we extract the derivatives outside the integral to reduce the degree of divergence of the terms remaining inside the integral. This requires to convert all primed derivatives into unprimed ones (so that they can be freely moved outside the integral.) The final results are expressed in terms of eleven external operators acting on the functions of y . The procedure of extracting derivatives involves a number of indefinite integration. We denote this operation by

$$I[f](y) \equiv \int^y dy' f(y'). \quad (68)$$

For example, the following identity derived in [15]

$$f(y) \left\{ A''(y) \frac{\partial y}{\partial x^\rho} \frac{\partial y}{\partial x'^\sigma} + A'(y) \frac{\partial^2 y}{\partial x^\rho \partial x'^\sigma} \right\} = \partial_\rho \partial'_\sigma I^2[fA''](y) + \frac{\partial^2 y}{\partial x^\rho \partial x'^\sigma} I[f'A'](y). \quad (69)$$

describes the operation: the expression in the left-hand-side is converted to derivatives acting a function of y (as an indefinite integral function of y) plus an extra function of y in the right-hand-side. Similar identities can be found in [15].

$$f(y)\partial_0\partial_0'A(y) = \partial_0\partial_0'I^2[fA''](y) - \frac{1}{2}\nabla\cdot\nabla'I^3[f'A'](y) + H^2aa'\left\{(2-y)I[f'A'](y) - (D-1)I^2[f'A'](y)\right\}, \quad (70)$$

$$f(y)\partial_0\partial_j'A(y) = \partial_0\partial_j'I^2[fA''](y) + Ha\partial_j'I^2[f'A'](y), \quad (71)$$

$$f(y)\partial_i\partial_0'A(y) = \partial_i\partial_0'I^2[fA''](y) + Ha'\partial_iI^2[f'A'](y), \quad (72)$$

$$f(y)\partial_i\partial_j'A(y) = \partial_i\partial_j'I^2[fA''](y) - 2H^2aa'\eta_{ij}I[f'A'](y). \quad (73)$$

Note that the derivatives are acting on functions of y externally in the above expressions. For our computation we need to derive these kind of expressions for a more complicated situation where there are four derivatives, spatial and temporal, acting on functions $A(y), B(y), C(y)$ that appear in equation (66),

$$F(y)\partial_\rho\partial_\sigma\partial'_\alpha\partial'_\beta A(y) = \partial_\rho\partial_\sigma\partial'_\alpha\partial'_\beta \left\{ I^4[FA''''](y) \right\} + \frac{\partial^4 y}{\partial x^\rho\partial x^\sigma\partial x'^\alpha\partial x'^\beta} \left\{ I[F'A'] + I^2[F'A''] + I^3[F'A'''] \right\} \\ + \left(2\frac{\partial y}{\partial x^{(\rho}} \frac{\partial^3 y}{\partial x^\sigma)\partial x'^\alpha\partial x'^\beta} + 2\frac{\partial y}{\partial x'^{(\alpha}} \frac{\partial^3 y}{\partial x'^\beta)\partial x^\rho\partial x^\sigma} + \frac{\partial^2 y}{\partial x^\rho\partial x^\sigma} \frac{\partial^2 y}{\partial x'^\alpha\partial x'^\beta} + 2\frac{\partial^2 y}{\partial x^\rho\partial x'^{(\alpha}} \frac{\partial^2 y}{\partial x'^\beta)\partial x^\sigma} \right) \left\{ I[F'A''] + I^2[F'A'''] \right\} \\ + \left(4\frac{\partial y}{\partial x^{(\rho}} \frac{\partial^2 y}{\partial x^\sigma)\partial x'^{(\alpha}} \frac{\partial y}{\partial x'^\beta)} + \frac{\partial y}{\partial x^\rho} \frac{\partial y}{\partial x^\sigma} \frac{\partial^2 y}{\partial x'^\alpha\partial x'^\beta} + \frac{\partial y}{\partial x'^\alpha} \frac{\partial y}{\partial x'^\beta} \frac{\partial^2 y}{\partial x^\rho\partial x^\sigma} \right) \left\{ I[F'A'''] \right\}. \quad (74)$$

Let us apply this method to the simplest cases, which is all of the derivatives are spatial, i.e. $F(y)\nabla^4A(y)$ or $F(y)\nabla^4C(y)$ terms in equation (66).

$$F(y)\nabla^4A(y) = \partial_i\partial_i\partial'_j\partial'_j \left\{ I^4[FA''''](y) \right\} + \frac{\partial^4 y}{\partial x^i\partial x^i\partial x'^j\partial x'^j} \left\{ I[F'A'] + I^2[F'A''] + I^3[F'A'''] \right\} \\ + \left(2\frac{\partial y}{\partial x^{(i}} \frac{\partial^3 y}{\partial x^i)\partial x'^j\partial x'^j} + 2\frac{\partial y}{\partial x'^{(j}} \frac{\partial^3 y}{\partial x'^j)\partial x^i\partial x^i} + \frac{\partial^2 y}{\partial x^i\partial x^i} \frac{\partial^2 y}{\partial x'^j\partial x'^j} + 2\frac{\partial^2 y}{\partial x^i\partial x'^{(j}} \frac{\partial^2 y}{\partial x'^j)\partial x^i} \right) \left\{ I[F'A''] + I^2[F'A'''] \right\} \\ + \left(4\frac{\partial y}{\partial x^{(i}} \frac{\partial^2 y}{\partial x^i)\partial x'^{(j}} \frac{\partial y}{\partial x'^j)} + \frac{\partial y}{\partial x^i} \frac{\partial y}{\partial x^i} \frac{\partial^2 y}{\partial x'^j\partial x'^j} + \frac{\partial y}{\partial x'^j} \frac{\partial y}{\partial x'^j} \frac{\partial^2 y}{\partial x^i\partial x^i} \right) \left\{ I[F'A'''] \right\}. \quad (75)$$

As one can see from the above equation we need to take up to four derivatives of y with respect to spatial variable x^i or x'^j and make use of equation (73). Using the following derivative identities

$$\begin{aligned} \frac{\partial y}{\partial x^i} &= 2H^2 aa' \Delta x_i, \quad \frac{\partial y}{\partial x'^j} = -2H^2 aa' \Delta x_j, \quad \frac{\partial^2 y}{\partial x^i \partial x^j} = 2H^2 aa' \eta_{ij}, \quad \frac{\partial^2 y}{\partial x^i \partial x'^j} = -2H^2 aa' \eta_{ij}, \\ \frac{\partial^2 y}{\partial x^i \partial x^i} &= \frac{\partial^2 y}{\partial x'^j \partial x'^j} = 2H^2 aa' (D-1), \quad \frac{\partial^3 y}{\partial x^i \partial x^i \partial x'^j} = \frac{\partial^3 y}{\partial x^i \partial x'^j \partial x'^j} = \frac{\partial^4 y}{\partial x^i \partial x^i \partial x'^j \partial x'^j} = 0. \end{aligned} \quad (76)$$

Eq. (75) will reduce into

$$\begin{aligned} F(y) \nabla^4 A(y) &= \nabla^4 \left\{ I^4 [F A''''](y) \right\} + \left(\left[2H^2 aa' (D-1) \right]^2 + 2 \left[2H^2 aa' \right]^2 (D-1) \right) \left\{ I[F' A''] + I^2[F' A'''] \right\} \\ &\quad + \left(4 \left[2H^2 aa' \right]^3 \Delta x_i \Delta x_j \eta_{ij} + \left[2H^2 aa' \right]^3 \Delta x_i \Delta x_i (D-1) + \left[2H^2 aa' \right]^3 \Delta x_i \Delta x_i (D-1) \right) \left\{ I[F' A'''] \right\} \\ &= \nabla^4 \left\{ I^4 [F A''''](y) \right\} + 8(D^2 - 1) \left[H^2 aa' \right]^2 \left\{ I[F' A''] + I^2[F' A'''] \right\} + 16(D+1) \left[H^2 aa' \right]^3 \|\Delta \vec{x}\|^2 \left\{ I[F' A'''] \right\}. \end{aligned} \quad (77)$$

The final form of (75) is:

$$F(y) \nabla^4 A(y) = \nabla^4 \left\{ I^4 [F A''''](y) \right\} + 4(D^2 - 1) H^4 (aa')^2 \left\{ I[F' A''] - I^2[F' A'''] \right\} + 4(D+1) H^2 aa' \nabla^2 I^3[F' A'''], \quad (78)$$

where the following identity was used to get the above desired form:

$$aa' H^2 \|\Delta \vec{x}\|^2 f(y) = -\frac{1}{2} (D-1) I[f](y) - \frac{\nabla \cdot \nabla'}{4aa' H^2} I^2[f](y). \quad (79)$$

This result is tabulated in Table V. Similar, but much more tedious work should be done to extract derivatives out for pure temporal and temporal spatial mixed derivatives in equation (66). These results are all tabulated in Tables VI, VII, VIII, in the form of the following eleven external operators acting on functions of y .

$$\alpha \equiv (aa')^2 \square^2, \quad (80)$$

$$\beta_1 \equiv (aa')^2 H^2 \square, \quad (81)$$

$$\beta_2 \equiv aa' (a^2 + a'^2) H^2 \square, \quad (82)$$

$$\gamma_1 \equiv (aa')^2 H^4, \quad (83)$$

$$\gamma_2 \equiv aa' (a^2 + a'^2) H^4, \quad (84)$$

$$\gamma_3 \equiv aa' (a + a')^2 H^4 = 2\gamma_1 + \gamma_2, \quad (85)$$

$$\delta \equiv (a^2 + a'^2) \nabla^2 \square, \quad (86)$$

$$\epsilon_1 \equiv aa' H^2 \nabla^2, \quad (87)$$

$$\epsilon_2 \equiv (a^2 + a'^2) H^2 \nabla^2, \quad (88)$$

$$\epsilon_3 \equiv (a + a')^2 H^2 \nabla^2 = 2\epsilon_1 + \epsilon_2, \quad (89)$$

$$\zeta \equiv \nabla^4. \quad (90)$$

The last step before completing this section is adding various components the 3-point contribution coming from $A(y)$, $B(y)$ and $C(y)$ in equation (66). The result can be written symbolically as

$$\begin{aligned} -iM_{3\text{pt}}^2(x; x') &= -\tilde{\kappa}^2 (aa')^{\frac{D}{2}-1} \left\{ \alpha f_\alpha(y) + \beta_1 f_{\beta_1}(y) + \beta_2 f_{\beta_2}(y) + \gamma_1 f_{\gamma_1}(y) + \gamma_2 f_{\gamma_2}(y) + \gamma_3 f_{\gamma_3}(y) \right. \\ &\quad \left. + \delta f_\delta(y) + \epsilon_1 f_{\epsilon_1}(y) + \epsilon_2 f_{\epsilon_2}(y) + \epsilon_3 f_{\epsilon_3}(y) + \zeta f_\zeta(y) \right\}. \end{aligned} \quad (91)$$

Here the functions, on which eleven external operators are acting, are given in the Tables IX, X, XI, XII, XIII, XIV, XV, XVI, XVII, XVIII, and XIX in Appendix B. This self-mass-squared will eventually be integrated over $d^4 x'$ in the effective field equation, (2). Thus, after extracting the derivative operators outside the integral we only need to retain D dimension for terms which diverge logarithmically¹ and higher at $x = x'$ for $D = 4$ in the coefficient functions, $f_i(y)$.

¹ Note that $y(x; x')$ vanishes like $(x - x')^2$ at coincidence and so $\int d^4 x' 1/y^2$ diverges logarithmically.

IV. RENORMALIZATION

In this section we renormalize the scalar self-mass-squared by subtracting counterterms depicted in Fig. 4. First, we construct counterterms applying the Bogoliubov, Parasiuk, Hepp and Zimmermann (BPHZ) scheme [45]. The structure of our Lagrangian of a scalar, conformally coupled to gravity, allows us to determine three de Sitter invariant counterterms at one loop order. On the other hand, our gauge fixing condition (35) breaks de Sitter symmetry, which results in possibility of having de Sitter noninvariant counterterms. It turns out that there is only one noninvariant counterterm because the de Sitter breaking occurs in a particular way. Identifying these possible counterterms is of great importance for checking correctness of the calculation. Hundreds of terms arising from various places should all add up to terms which respect the symmetries not broken due to the gauge fixing term. We will soon show this occurs in a highly nontrivial way. The next step is to collect all the divergences occurred in the previous section and localize them (in the form of δ -function) so as to be absorbed by the local counterterms. Finally, we obtain a finite result which can be used to solve the effective field equation (2) at one loop order. This procedure is summarized in the following two subsections.

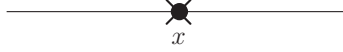


FIG. 4: Contribution from counterterms.

A. Construction of counterterms

To construct possible counterterms we first note that the superficial degree of divergence for the scalar-graviton interaction in our Lagrangian (3) at one loop order is four. This means that in order to cancel those divergences, the corresponding counterterms should have a mass dimension of four. There are also two basic requirements for our case. One is, the counterterms must carry two scalar fields, each of which counts one dimension of mass. The other is, they should also contain one factor of the loop counting parameter κ^2 which has the dimension of $mass^{-2}$. We therefore require each counterterm to have an additional mass dimension of four. Because our scalar is massless, this can only be achieved by carrying four derivatives. There are three ways to form an invariant satisfying these requirements.

- All four derivatives act on scalars.
- Two derivatives act on scalars and the other two act on the metric.
- All four derivatives act on the metric.

Applying these three ways, we find five invariants listed as follows:

$$\kappa^2 \varphi_{;\mu\nu} \varphi_{;\rho\sigma} g^{\mu\nu} g^{\rho\sigma} \sqrt{-g}, \quad \kappa^2 \partial_\mu \varphi \partial_\nu \varphi R g^{\mu\nu} \sqrt{-g}, \quad \kappa^2 \partial_\mu \varphi \partial_\nu \varphi R^{\mu\nu} \sqrt{-g}, \quad \kappa^2 \varphi^2 R^2 \sqrt{-g} \quad \text{and} \quad \kappa^2 \varphi^2 R^{\mu\nu} R_{\mu\nu} \sqrt{-g}. \quad (92)$$

Finally, specializing to the de Sitter background, i.e., $\hat{R}_{\mu\nu} = (D-1)H^2 \hat{g}_{\mu\nu}$, allows us to put the second and third of (92) together and the fourth and fifth together into one term. This results in three invariant counterterms using the rescaled field, $\tilde{\phi} \equiv \phi = a\varphi$, see Eq. (16),

$$\Delta\mathcal{L}_1 = \frac{1}{2}c_1\kappa^2\Box\phi\Box\phi a^2, \quad \Delta\mathcal{L}_2 = -\frac{1}{2}c_2\kappa^2 H^2 \partial_\mu \phi \partial^\mu \phi, \quad \Delta\mathcal{L}_3 = \frac{1}{2}c_3\kappa^2 H^4 \phi^2 a^2. \quad (93)$$

The vertices associated to these counterterms are,

$$\left. \frac{i\delta\Delta S_1}{\delta\phi(x)\delta\phi(x')} \right|_{\phi=0} = i c_1 \kappa^2 a^2 \Box^2 \delta^D(x-x'), \quad (94)$$

$$\left. \frac{i\delta\Delta S_2}{\delta\phi(x)\delta\phi(x')} \right|_{\phi=0} = i c_2 \kappa^2 H^2 a^2 \Box \delta^D(x-x'), \quad (95)$$

$$\left. \frac{i\delta\Delta S_3}{\delta\phi(x)\delta\phi(x')} \right|_{\phi=0} = i c_3 \kappa^2 H^4 a^2 \delta^D(x-x'). \quad (96)$$

Here the coefficients c_i will be determined by requirement of canceling the divergences.

The de Sitter noninvariant counterterm drawn to attention in the beginning of this section was constructed in Ref. [15] by carefully examining which symmetries are broken by our gauge choice (35). Here we review essential points of the construction strategy taken by [15] and finally give the unique noninvariant counterterm. The first point is that our gauge fixing term breaks only spatial special conformal transformations among the full $\frac{1}{2}D(D+1)$ de Sitter isometries:

- $(D-1)$ spatial translations :

$$\eta' = \eta, \quad x'^i = x^i + \epsilon^i. \quad (97)$$

- $\frac{1}{2}(D-1)(D-2)$ rotations :

$$\eta' = \eta, \quad x'^i = R^{ij}x^j. \quad (98)$$

- 1 dilation :

$$\eta' = k\eta, \quad x'^i = kx^i. \quad (99)$$

- $(D-1)$ spatial special conformal transformations :

$$\eta' = \frac{\eta}{1 - 2\vec{\theta} \cdot \vec{x} + \|\vec{\theta}\|^2 x \cdot x}, \quad x'^i = \frac{x^i - \theta^i x \cdot x}{1 - 2\vec{\theta} \cdot \vec{x} + \|\vec{\theta}\|^2 x \cdot x}. \quad (100)$$

Hence we can use the residual symmetries respected by our gauge condition to restrict the form of noninvariant counterterms. Homogeneity (97) requires that the counterterm cannot depend on the spatial coordinates x^i . Isotropy (98) implies that spatial derivative operators ∂_i must be contracted into each another. Dilation symmetry (99) restricts that derivative operators and the conformal time η can only appear in the form $a^{-1}\partial_\mu$. These constraints and a number of partial integration lead us to three possible noninvariant counterterms,

$$\frac{1}{2}\kappa^2 a^{D-2} \square \varphi \nabla^2 \varphi, \quad \frac{1}{2}\kappa^2 a^{D-4} \nabla^2 \varphi \nabla^2 \varphi \quad \text{and} \quad -\frac{1}{2}\kappa^2 H^2 a^{D-2} \nabla \varphi \cdot \nabla \varphi. \quad (101)$$

Finally we note that our gauge fixing term (35) becomes Poincaré invariant in the flat space limit of $H \rightarrow 0$, where the conformal time $\eta = -e^{-Ht}/H$ with the comoving time t held fixed. Only the final term of (101) vanishes in the flat space limit and it can serve as a noninvariant counterterm. Using the rescaled field it becomes

$$\Delta \mathcal{L}_4 = -\frac{1}{2}\kappa^2 H^2 \nabla \phi \cdot \nabla \phi, \quad (102)$$

with the associated vertex,

$$\left. \frac{i\delta \Delta S_4}{\delta \phi(x)\delta \phi(x')} \right|_{\phi=0} = i c_4 \kappa^2 H^2 \nabla^2 \delta^D(x-x'). \quad (103)$$

To summarize, there are only four - three invariant and one noninvariant - counterterms allowed for the CC scalar self-mass-squared calculation at one loop order. Any occurrences of divergent terms which do not fit in one of these four counterterms would immediately imply errors in the calculation. Again we emphasize that this provides with a crucial check for perturbative quantum gravity computations like our current work. It is amazing to observe that divergences of the “non-permitted” form occurring in various places add up to zero in the first column of TABLE III. Note that the second column becomes finite for $D=4$ by canceling the overall divergent factor $1/(D-4)$.

B. Localization of divergences and a finite result

Now with the prepared counterterms, it is time to collect all the divergent terms and segregate them into a local function in the form of the counterterm vertices, that is, an operator acting on the $\delta^D(x-x')$. Recall that the

contributions from 4-point interactions are already finite and the divergences from 3-point vertices involve powers of y that are not integrable for $D = 4$ dimensions,

$$\left(\frac{4}{y}\right)^D, \left(\frac{4}{y}\right)^{D-1} \quad \text{and} \quad \left(\frac{4}{y}\right)^{D-2}. \quad (104)$$

Again we use the technique of extracting derivative operators to make the terms less singular. Specifically we use the following identity to extract d' Alembertian operators from these terms until they become integrable,

$$\square f(y) = H^2 \left[(4y - y^2) f''(y) + D(2 - y) f'(y) \right] + \text{Res} \left[y^{\frac{D}{2}-2} f \right] \frac{4\pi^{D/2} H^{2-D}}{\Gamma(\frac{D}{2} - 1)} \frac{i}{\sqrt{-g}} \delta^D(x - x'). \quad (105)$$

Here $\text{Res}[F]$ means the residue of $F(y)$. Applying this identity (105) each of the nonintegrable terms becomes,

$$\left(\frac{4}{y}\right)^D = \frac{2}{(D-1)D} \frac{\square}{H^2} \left(\frac{4}{y}\right)^{D-1}, \quad (106)$$

$$\left(\frac{4}{y}\right)^{D-1} = \frac{2}{(D-2)^2} \frac{\square}{H^2} \left(\frac{4}{y}\right)^{D-2} - \frac{2}{D-2} \left(\frac{4}{y}\right)^{D-2}, \quad (107)$$

$$\left(\frac{4}{y}\right)^{D-2} = \frac{2}{(D-4)(D-3)} \frac{\square}{H^2} \left(\frac{4}{y}\right)^{D-3} - \frac{4}{D-4} \left(\frac{4}{y}\right)^{D-3}. \quad (108)$$

Note that the logarithmically divergent power $1/y^{D-2}$ in (108) has reduced to the power $1/y^{D-3}$ which is integrable, however it has a divergent factor of $1/(D-4)$. Applying the key identity (105) to the power of $1/y^{D/2-1}$ we have zero in the form,

$$0 = \frac{\square}{H^2} \left(\frac{4}{y}\right)^{\frac{D}{2}-1} - \frac{D}{2} \left(\frac{D}{2} - 1\right) \left(\frac{4}{y}\right)^{\frac{D}{2}-1} - \frac{(4\pi)^{D/2} H^{-D}}{\Gamma(\frac{D}{2} - 1)} \frac{i}{a^D} \delta^D(x - x'). \quad (109)$$

Adding this expression of zero (109) to (108) allows us to segregate the divergence on the local term,

$$\begin{aligned} \left(\frac{4}{y}\right)^{D-2} &= \frac{2}{(D-4)(D-3)} \left\{ \frac{(4\pi)^{\frac{D}{2}} H^{-D}}{\Gamma(\frac{D}{2} - 1)} \frac{i \delta^D(x - x')}{a^D} + \frac{\square}{H^2} \left[\left(\frac{4}{y}\right)^{D-3} - \left(\frac{4}{y}\right)^{\frac{D}{2}-1} \right] \right\} \\ &\quad - \frac{4}{D-4} \left\{ \left(\frac{4}{y}\right)^{D-3} - \frac{D(D-2)}{8(D-3)} \left(\frac{4}{y}\right)^{\frac{D}{2}-1} \right\}, \end{aligned} \quad (110)$$

$$= \frac{i H^{-D} (4\pi)^{\frac{D}{2}}}{(D-4)(D-3)\Gamma(\frac{D}{2})} (D-2) \frac{\delta^D(x - x')}{a^D} - \frac{\square}{H^2} \left\{ \frac{4}{y} \ln\left(\frac{y}{4}\right) \right\} + 2 \left(\frac{4}{y}\right) \ln\left(\frac{y}{4}\right) - \left(\frac{4}{y}\right) + \mathcal{O}(D-4), \quad (111)$$

or

$$\begin{aligned} \left(\frac{1}{y}\right)^{D-2} &= \frac{i H^{-D} \pi^{\frac{D}{2}}}{(D-4)(D-3)\Gamma(\frac{D}{2})4^{\frac{D}{2}}} 16(D-2) \frac{\delta^D(x - x')}{a^D} \\ &\quad - \frac{16}{4^D} \frac{\square}{H^2} \left\{ \frac{4}{y} \ln\left(\frac{y}{4}\right) \right\} + \frac{32}{4^D} \left(\frac{4}{y}\right) \ln\left(\frac{y}{4}\right) - \frac{16}{4^D} \left(\frac{4}{y}\right) + \mathcal{O}(D-4). \end{aligned} \quad (112)$$

Substituting (111) into (107) the quadratically divergent power becomes,

$$\begin{aligned} \left(\frac{1}{y}\right)^{D-1} &= \frac{i H^{-D} \pi^{\frac{D}{2}}}{(D-4)(D-3)\Gamma(\frac{D}{2})4^{\frac{D}{2}}} \left\{ \frac{8}{D-2} \frac{\square}{H^2} - 8 \right\} \frac{\delta^D(x - x')}{a^D} - \frac{2}{4^D} \frac{\square^2}{H^4} \left\{ \frac{4}{y} \ln\left(\frac{y}{4}\right) \right\} \\ &\quad + \frac{\square}{H^2} \left\{ \frac{8}{4^D} \left(\frac{4}{y}\right) \ln\left(\frac{y}{4}\right) - \frac{2}{4^D} \left(\frac{4}{y}\right) \right\} - \frac{8}{4^D} \left(\frac{4}{y}\right) \ln\left(\frac{y}{4}\right) + \frac{4}{4^D} \left(\frac{4}{y}\right) + \mathcal{O}(D-4). \end{aligned} \quad (113)$$

Similarly, the quartically divergent term becomes,

$$\begin{aligned} \left(\frac{1}{y}\right)^D &= \frac{i H^{-D} \pi^{\frac{D}{2}}}{(D-4)(D-3)\Gamma(\frac{D}{2})4^{\frac{D}{2}}} \left\{ \frac{4}{(D-2)(D-1)D} \frac{\square^2}{H^2} - \frac{4}{(D-1)D} \frac{\square}{H^2} \right\} \frac{\delta^D(x - x')}{a^D} \\ &\quad - \frac{1}{4^D} \left\{ \frac{1}{12} \frac{\square^3}{H^6} \left[\frac{4}{y} \ln\left(\frac{y}{4}\right) \right] + \frac{\square^2}{H^4} \left[\frac{1}{3} \left(\frac{4}{y}\right) \ln\left(\frac{y}{4}\right) - \frac{1}{12} \left(\frac{4}{y}\right) \right] - \frac{1}{3} \left(\frac{4}{y}\right) \ln\left(\frac{y}{4}\right) + \frac{1}{6} \left(\frac{4}{y}\right) \right\} + \mathcal{O}(D-4). \end{aligned} \quad (114)$$

Plugging these expressions into (91) would make the divergent pieces almost look like the local counterterms except d'Alembertians acting on both powers of a and $\delta^D(x-x')$. We employ the following identities to pass all factors of a to the left.

$$\square a^n \delta^D(x-x') = \left(a^n \square - [n^2 + n(D-1)]H^2 a^n - 2nHa^{n-1}\partial_0 \right) \delta^D(x-x'), \quad (115)$$

$$\square^2 a^n \delta^D(x-x') = \left(a^n \square^2 - 2n[3n + (D-1)]H^2 a^n \square - 4nHa^{n-1}\partial_0 \square + 4n^3 H^3 a^{n-1}\partial_0 \right. \\ \left. + 4n(n-1)H^2 a^{n-2}\nabla^2 + n^2[n + (D-1)]^2 H^4 a^n \right) \delta^D(x-x'). \quad (116)$$

The following example for the term with the external operator α shows how we use these identities in order to put the divergent pieces in the form of the counterterms

$$(aa')^{\frac{D}{2}-1} \alpha \left\{ -\frac{(D+2)(D+4)H^{2D-4}\Gamma(\frac{D}{2}-1)^2}{128(D-2)(D+1)\pi^D} \left(\frac{1}{y}\right)^{D-2} \right\} \\ = -\frac{(D+2)(D+4)H^{2D-4}\Gamma(\frac{D}{2}-1)^2}{128(D-2)(D+1)\pi^D} (aa')^{\frac{D}{2}+1} \square^2 \left(\frac{1}{y}\right)^{D-2}, \quad (117)$$

$$= -\frac{(D+2)(D+4)H^{2D-4}\Gamma(\frac{D}{2}-1)^2}{128(D-2)(D+1)\pi^D} \frac{iH^{-D}\pi^{\frac{D}{2}}}{(D-4)(D-3)\Gamma(\frac{D}{2})4^{\frac{D}{2}}} a^{\frac{D}{2}+1} 16(D-2) \square^2 a^{\frac{D}{2}+1} \frac{\delta^D(x-x')}{a^D}, \quad (118)$$

$$= -\frac{(D+2)(D+4)}{2^5(D-2)^3(D+1)} \frac{iH^{D-4}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)} \left\{ 16(D-2)a^2 \square^2 - 8(D-2)^2(D-4)H^2 a^2 \square \right. \\ \left. + 32(D-2)^2 Ha \partial_0 \square - 8(D-2)^4 H^3 a \partial_0 + 16(D-2)DH^2 \nabla^2 - (D-2)^2 D^2 H^4 a^2 \right\} \delta^D(x-x'). \quad (119)$$

Here α is given in (80). Note that the first, second, fifth and sixth terms correspond to the counterterm vertices (94), (95), (103), and (96), respectively.

Using the same procedure for the terms with the remaining ten external operators, we can segregate all divergent terms into the form which derivatives acting on $\delta^D(x-x')$. The results are given in TABLE I, TABLE II and TABLE III. Note that we have another consistency check for the calculation: the contribution to the counterterm (94) vanishes as it must, otherwise this counterterm would not be zero in flat space and break Poincaré invariance.

TABLE I: Contributions to counterterms. All terms are multiplied by $\frac{i\kappa^2 H^{D-4}}{(4\pi)^{D/2}} \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)}$

External operators	Coef. of $H^4 a^2 \delta^D(x-x')$
α	$\frac{D^2(D+2)(D+4)}{32(D-2)(D+1)}$
β_1	$\frac{(D-1)D(D+2)(D+4)}{8(D+1)}$
β_2	$-\frac{(D+2)(D+4)(D^2-2D-4)}{8(D+1)}$
γ_1	$\frac{D(D^5-35D^4+67D^3-2D^2-264D-64)}{32(D-2)(D+1)}$
γ_2	$\frac{2D^6+35D^5-108D^4-144D^3+384D+704}{8(D-2)^2(D+1)}$
Total	$\frac{D^7-29D^6+282D^5-540D^4-856D^3+224D^2+1664D+3328}{32(D-2)^2(D+1)}$

Besides the local divergent terms, the remaining pieces are nonlocal and finite for $D = 4$ dimension. Again we

TABLE II: Contributions to counterterms. All terms are multiplied by $\frac{i\tilde{\kappa}^2 H^{D-4}}{(4\pi)^{D/2}} \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)}$.

External operators	Coef. of $a^2 \square^2 \delta^D(x-x')$	Coef. of $H^2 a^2 \square \delta^D(x-x')$	Coef. of $H^2 \nabla^2 \delta^D(x-x')$
α	$-\frac{(D+2)(D+4)}{2(D-2)^2(D+1)}$	$\frac{(D-4)(D+2)(D+4)}{4(D-2)(D+1)}$	$-\frac{D(D+2)(D+4)}{2(D-2)(D+1)}$
β_1	0	$\frac{(D-1)(D+2)(D+4)}{2(D-2)(D+1)}$	0
β_2	0	$-\frac{(D+2)(D+4)}{2(D+1)}$	0
γ_1	$\frac{(D-20)(D+2)}{2(D-2)^2(D+1)}$	$\frac{D^5-37D^4+116D^3-172D^2-496D+576}{8(D-2)^2(D+1)}$	$\frac{(D-20)D(D+2)}{2(D-2)(D+1)}$
γ_2	$\frac{12(D+2)}{(D-2)^2(D+1)}$	$\frac{D^4+12D^3-5D^2-40D-184}{(D-2)^2(D+1)}$	$\frac{12(D^3+8)}{(D-2)^2(D+1)}$
ϵ_1	0	0	$\frac{D^4+12D^3-24D^2-20D+16}{(D-2)(D^2-1)}$
ϵ_2	0	0	$-\frac{13D^2+6D-16}{(D^2-1)}$
ϵ_3	0	0	$\frac{2(D-2)(D+2)(D+4)}{(D^2-1)}$
Total	0	$\frac{(D-4)(D^4-23D^3+124D^2+260D+208)}{8(D-2)^2(D+1)}$	$\frac{3D^5-3D^4-34D^3+96D^2+112D-192}{(D-2)^2(D^2-1)}$

TABLE III: Other contributions to counterterms. All terms are multiplied by $\frac{i\tilde{\kappa}^2 H^{D-4}}{(4\pi)^{D/2}} \frac{\Gamma(\frac{D}{2})}{(D-4)(D-3)}$.

External operators	Coef. of $Ha\partial_0 \square \delta^D(x-x')$	Coef. of $H^3 a\partial_0 \delta^D(x-x')$
α	$-\frac{(D+2)(D+4)}{(D-2)(D+1)}$	$\frac{(D-2)(D+2)(D+4)}{4(D+1)}$
β_1	0	$\frac{(D-1)(D+2)(D+4)}{2(D+1)}$
β_2	0	$-\frac{(D-2)(D+2)(D+4)}{2(D+1)}$
γ_1	$\frac{(D-20)(D+2)}{(D-2)(D+1)}$	$\frac{D^5-37D^4+112D^3-92D^2-480D+256}{8(D-2)(D+1)}$
γ_2	$\frac{24(D+2)}{(D-2)(D+1)}$	$\frac{D^4+12D^3-5D^2-64D-232}{(D-2)(D+1)}$
Total	0	$\frac{(D-4)(D^4-23D^3+124D^2+356D+400)}{8(D-2)(D+1)}$

illustrate how the nonlocal finite terms are identified using the case with the α operator.

$$\begin{aligned} & (aa')^{\frac{D}{2}-1} \alpha \left\{ -\frac{(D+2)(D+4)H^{2D-4}\Gamma(\frac{D}{2}-1)^2}{128(D-2)(D+1)\pi^D} \left(\frac{1}{y}\right)^{D-2} - \frac{3H^4}{32\pi^4} \frac{1}{y} \right\} \\ &= -\frac{(D+2)(D+4)H^{2D-4}\Gamma(\frac{D}{2}-1)^2}{128(D-2)(D+1)\pi^D} (aa')^{\frac{D}{2}+1} \square^2 \left(\frac{1}{y}\right)^{D-2} - (aa')^{\frac{D}{2}+1} \square^2 \frac{3H^4}{32\pi^4} \frac{1}{y}, \end{aligned} \quad (120)$$

$$\begin{aligned} &= -\frac{(D+2)(D+4)H^{2D-4}\Gamma(\frac{D}{2}-1)^2}{128(D-2)(D+1)\pi^D} (aa')^{\frac{D}{2}+1} \square^2 \left\{ -\frac{16}{4^D} \frac{\square}{H^2} \left\{ \frac{4}{y} \ln\left(\frac{y}{4}\right) \right\} + \frac{32}{4^D} \left(\frac{4}{y}\right) \ln\left(\frac{y}{4}\right) - \frac{16}{4^D} \left(\frac{4}{y}\right) \right\}, \\ &\quad - (aa')^{\frac{D}{2}+1} \square^2 \frac{3H^4}{32\pi^4} \left(\frac{1}{y}\right), \end{aligned} \quad (121)$$

$$\begin{aligned} &= -\frac{3}{80} \frac{H^4}{\pi^4} (aa')^3 \square^2 \left\{ -\frac{16}{4^4} \frac{\square}{H^2} \left(\frac{\ln x}{x}\right) + \frac{32}{4^4} \left(\frac{\ln x}{x}\right) - \frac{16}{4^4} \left(\frac{1}{x}\right) \right\} - (aa')^3 \square^2 \frac{3H^4}{32\pi^4} \left(\frac{1}{4x}\right), \\ &= \frac{3}{1280} \frac{H^2}{\pi^4} (aa')^3 \square^3 \left(\frac{\ln x}{x}\right) - \frac{3}{640} \frac{H^4}{\pi^4} (aa')^3 \square^2 \left(\frac{\ln x}{x} + \frac{9}{2x}\right). \end{aligned} \quad (122)$$

This time we only write the nonlocal finite pieces without $\delta^D(x-x')$ from the power $(\frac{1}{y})^{D-2}$ in (112). In the final two lines, we re-define the function $y(x; x')$ in (33) as $x \equiv \frac{y}{4}$. Also, note that we take the $D=4$ limit to get the final result. Using the same method for the terms with the other external operators we obtain nonlocal finite terms for each case, i.e., each external operator. These newly found finite terms are added to the already found finite terms listed in TABLE IX - XIX. The summation of all the finite nonlocal terms from the 3-point interactions are given in TABLE IV.

TABLE IV: All Finite Nonlocal Contributions with $x \equiv \frac{y}{4}$, where $y(x; x')$ is defined in the equation (33).

External operators	Coef. of $\frac{\tilde{\kappa}^2 H^4}{(4\pi)^4}$
$(aa')^3 \square^3 / H^2$	$\frac{9 \ln x}{5x}$
$(aa')^3 \square^2$	$\frac{9 \ln x}{x} - \frac{21}{5x}$
$(aa')^3 H^2 \square$	$-\frac{267 \ln x}{5x} + \frac{51}{x}$
$(aa')^3 H^4$	$\frac{258 \ln x}{5x} - \frac{549}{5x}$
$(aa')^2 (a^2 + a'^2) \square^3 / H^2$	$-\frac{9 \ln x}{10x}$
$(aa')^2 (a^2 + a'^2) \square^2$	$-\frac{99 \ln x}{10x} - \frac{9}{10x}$
$(aa')^2 (a^2 + a'^2) H^2 \square$	$\frac{36 \ln x}{x} - \frac{51}{2x}$
$(aa')^2 (a^2 + a'^2) H^4$	$-\frac{108 \ln x}{5x} + \frac{174}{5x}$
$(aa')^2 H^2 \nabla^2$	$\frac{128 \ln x}{5x} + \frac{166}{5x}$
$(aa')^2 \nabla^2 \square$	$-\frac{64 \ln x}{5x}$
$(aa')(a^2 + a'^2) H^2 \nabla^2$	$-\frac{4 \ln x}{x} - \frac{2}{x}$
$(aa')(a^2 + a'^2) \nabla^2 \square$	$\frac{2 \ln x}{x} + \frac{2}{x}$
$(aa') \nabla^4$	0

Our final result for the regulated self-mass-squared consists of three finite parts:

- the local 4-point contributions given in (59),
- the local 3-point contributions coming from TABLE I - III,

- the nonlocal pieces coming from 3-point interaction, i.e. TABLE IV,

It has the following form,

$$\begin{aligned}
-iM_{\text{reg}}^2(x; x') &= -i\kappa^2 a^2 \left(d_1 \square^2 + d_2 H^2 \square + d_3 H^4 + d_4 H^2 \frac{\nabla^2}{a^2} \right) \delta^D(x - x') \\
&+ \text{Table IV} + 2i\kappa^2 \times \frac{H^4 a^2}{16\pi^2} \delta^4(x - x') + \frac{9i\kappa^2}{40} \times \frac{H^3 a}{16\pi^2} \partial_0 \delta^4(x - x') + \mathcal{O}(D - 4) .
\end{aligned} \tag{123}$$

The last term in the above equation comes from the second column of Table III. Here the coefficients d_i are

$$d_1 = 0 , \tag{124}$$

$$d_2 = \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left[\frac{7}{80} + \mathcal{O}(D - 4) \right] , \tag{125}$$

$$\begin{aligned}
d_3 &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left[\frac{(D^7 - 29D^6 + 282D^5 - 540D^4 - 856D^3 + 224D^2 + 1664D + 3328)\Gamma(\frac{D}{2})}{2^{11}(D-4)(D-3)(D-1)^2(D+1)} \right] , \\
&= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left[\frac{3}{40(D-4)} + \frac{1021}{3600} - \frac{3\gamma}{80} + \mathcal{O}(D - 4) \right] ,
\end{aligned} \tag{126}$$

$$\begin{aligned}
d_4 &= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left[\frac{(3D^5 - 3D^4 - 34D^3 + 96D^2 + 112D - 192)\Gamma(\frac{D}{2})}{2^6(D-4)(D-3)(D-1)^2(D^2-1)} \right] , \\
&= \frac{H^{D-4}}{(4\pi)^{\frac{D}{2}}} \left[\frac{2}{9(D-4)} - \frac{59}{540} - \frac{\gamma}{9} + \mathcal{O}(D - 4) \right] .
\end{aligned} \tag{127}$$

Here γ is Euler's constant which is approximately equal to 0.577215. It appears here due to the expansion of the Gamma function. Following the BPHZ renormalization scheme, we will choose c_i 's to absorb the divergent terms:

$$c_i = -d_i + \Delta c_i \tag{128}$$

where Δc_i is the remaining arbitrary finite term for each of the four c_i . Taking the unregulated limit ($D=4$) provides us with the final renormalized result for the conformal-conformal interaction part of the full self-mass-squared at one loop order,

$$\begin{aligned}
-iM_{\text{ren}}^2(x; x') &= -i\kappa^2 a^2 \left(\Delta c_1 \square^2 + \Delta c_2 H^2 \square + \Delta c_3 H^4 + \Delta c_4 H^2 \frac{\nabla^2}{a^2} \right) \delta^4(x - x') \\
&+ \text{Table IV} + 2i\kappa^2 \times \frac{H^4 a^2}{16\pi^2} \delta^4(x - x') + \frac{9i\kappa^2}{40} \times \frac{H^3 a}{16\pi^2} \partial_0 \delta^4(x - x') + \mathcal{O}(D - 4) .
\end{aligned} \tag{129}$$

V. DISCUSSION

We have calculated the self-mass-squared of a conformally coupled scalar interacting with a graviton at one loop order on a locally de Sitter geometry. A technical advantage of the scalar being conformally coupled to gravity is that we can do the computations around the flat space metric. Specifically, we can use the conformally rescaled scalar propagator for flat space. On the other hand, the graviton propagator stays the same because it is not conformally invariant. The computation was done using dimensional regularization and renormalized by absorbing divergences with BPHZ counterterms. The fully renormalized result is given in Eq. (129) with the finite nonlocal contributions in TABLE IV.

The purpose of this computation is to investigate quantum gravitational effects to the scalar mode function during Inflation. Despite the fact that our scalar is taken to be a spectator field during inflation driven by cosmological constant Λ , one might still apply the result to the inflaton of scalar-driven inflation because its potentials are considered to be nearly flat. The first step for checking whether or not gravitons give a significant correction to scalars is to compute and renormalize the scalar self-mass-squared $-iM^2(x; x')$, which concerns our present paper. The second step is to solve the quantum corrected effective field equation (2). At this point it should be noted that this field equation (2) is derived in the in-out formalism which one typically uses for scattering amplitude computations in flat space. However, in a time-dependent background such as de Sitter, the in-vacuum does not evolve to the out-vacuum

and thus the in-out formalism is not applicable to our case. Therefore we will instead use the in-in formalism also called the Schwinger-Keldysh formalism [46–53] in which the effective field equation (2) is replaced by

$$a^4 \square \phi(x) - \frac{1}{6} R \phi(x) - \int_{\eta_i}^0 d\eta' \int d^3 x' \left\{ M_{++}^2(x; x') + M_{+-}^2(x; x') \right\} \phi(x') = 0. \quad (130)$$

Here in order to get $-iM_{++}^2(x; x')$ and $-iM_{+-}^2(x; x')$ (which are composed of $i\Delta_{++}(x; x')$ and $i\Delta_{+-}(x; x')$ respectively), we make the following substitutions for $y(x; x')$,

$$i\Delta_{++}(x; x') : \quad y \rightarrow y_{++}(x; x') \equiv a(\eta)a(\eta')H^2 \left[\|\vec{x} - \vec{x}'\|^2 - (|\eta - \eta'| - i\delta)^2 \right], \quad (131)$$

$$i\Delta_{+-}(x; x') : \quad y \rightarrow y_{+-}(x; x') \equiv a(\eta)a(\eta')H^2 \left[\|\vec{x} - \vec{x}'\|^2 - (\eta - \eta' + i\delta)^2 \right]. \quad (132)$$

Note that $-iM_{++}^2(x; x')$ is the same as the usual in-out self-mass-squared $-iM^2(x; x')$.

Our aim is to determine if the tree order mode function of the CC scalar (see for example, [54])

$$\phi_0(t, \vec{x}) = u(t, k) e^{i\vec{k} \cdot \vec{x}}, \quad \text{where} \quad u(t, k) = \sqrt{\frac{\hbar}{2ck}} \frac{\exp \left[-ick \int_t^{t_i} \frac{dt'}{a(t')} \right]}{a(t)}, \quad (133)$$

gets corrections from the self-mass-squared at one loop order, which will be a future paper.

The conformally coupled scalar field interacts with gravity via a kinetic term as well as the conformal coupling term. The case with no conformal coupling, which is called minimal coupling, was studied in the previous work [15]. In this paper we examined the other one, which is conformal coupling. For the case of 4-point interactions depicted in Fig. 2, adding these two results will suffice. However, for the case of 3 point interactions graphically represented in Fig. 3, one has the possibility of mixing the two. One vertex can be a kinetic interaction and the other can come from the conformal coupling. In the current work, we have completed only the half of the first step, namely calculating the one loop scalar self-mass-squared from conformal coupling. In a subsequent paper we will include the purely kinetic terms [15] and the mixing of kinetic and conformal interactions in Eq. (29) and use these full results in order to solve the effective field equation (130).

We also would like to highlight a couple of points to corroborate correctness of our calculation, which is a question that comes into mind for any computation of this length. The first one is that the most divergent part of the graviton propagator (which consists of the A, B and C parts) is same as the conformally coupled scalar propagator as one can see from Eqs. (48)-(50). Adding the tensor factors of the A, B and C parts of the graviton propagator gives

$$2\bar{\eta}_{\mu(\rho}\bar{\eta}_{\sigma)\nu} - \frac{2\bar{\eta}_{\mu\nu}\bar{\eta}_{\rho\sigma}}{D-3} - 4\delta_{(\mu}^0\bar{\eta}_{\nu)(\rho}\delta_{\sigma)}^0 + \frac{2[(D-3)\delta_{\mu}^0\delta_{\nu}^0 + \bar{\eta}_{\mu\nu}][(D-3)\delta_{\rho}^0\delta_{\sigma}^0 + \bar{\eta}_{\rho\sigma}]}{(D-3)(D-2)} = 2\eta_{\mu(\rho}\eta_{\sigma)\nu} - \frac{2}{D-2}\eta_{\mu\nu}\eta_{\rho\sigma}, \quad (134)$$

which is the tensor factor of the graviton propagator in flat space. This means that the most singular terms in the full computation are identical to those of flat space. This is why the sum of the coefficients for $a^2 \square^2 \delta^D(x - x')$ becomes zero in TABLE II. The second, and much stronger, check comes from the analysis of the de Sitter noninvariant counterterms. The de Sitter symmetry is broken, due to the form of our gauge fixing term, in such a way only a subgroup of the full de Sitter group is preserved. Respecting the remaining symmetries turns out to make the divergent terms $Ha\partial_0 \square \delta^D(x - x')$ and $H^3 a \partial_0 \delta^D(x - x')$ become zero. This is exactly what happens in TABLE III in a highly nontrivial way. And finally the last check is getting zero for ∇^4 in TABLE IV. Since working with the most singular term is similar to working in flat space where the gauge fixing term vanishes, one can at most get quadratic divergence. Therefore the de Sitter breaking gauge fixing term can only occur at quadratically divergent level. This will suffice to give only ∇^2 which breaks de Sitter invariance but still respects the other symmetries that are not broken.

One point worth to note is that the result for the minimal coupling only. The massless, minimally coupled scalar gets no significant corrections from inflationary gravitons at one loop order [15, 16]. That reason why is that, even though inflation produces the vast ensemble of gravitons, they only interact with the MMC scalar via kinetic energy which redshifts to zero at late times. Adding conformal coupling engenders another interaction, which is non-derivative and so not redshifted. There is clearly one term that might give a big effect, which is the $H^4 a^2 \delta^4(x - x')$ term in Eq. (129). This term was not present in the MMC calculation studied in [15, 16]. But, unfortunately this can be removed by the conformal counterterm, $a^2 \Delta c_3 H^4 \delta^4(x - x')$ with an arbitrary constant Δc_3 in front. However the finite terms we got here may give some enhancement after we solve the effective field equation. In [16] the corresponding finite terms nontrivially added up to be zero but there is no a priori reason why they should add up to zero at the end in

this calculation. Also including the MMC-CC cross terms might give an interesting effect and checking it explicitly is our goal for the series of this and subsequent papers.

Finally the most interesting and cosmologically relevant part would be the second step of solving the quantum corrected effective field equation for the scalar mode function. If the mode function gets quantum corrections, so does the power spectrum. Weinberg's analysis [53] suggests that the possibility of quantum loop corrections to the power spectra of primordial density perturbations. Our case, if we find any correction, would serve as a specific example for his analysis. In the age of possibility of observing primordial gravitational waves with a detector like BICEP2 [7], resolving quantum corrections due to gravitons, even though it may still take a few decades more, does not sound utterly hopeless. We hope to provide a foreground for those future measurements.

VI. ACKNOWLEDGMENT

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Appendix A: Tables for Identities to extract derivatives

TABLE V: For $F(y)\nabla^4 A(y)$.

External operators	The functions they act on
γ_1	$4(D^2 - 1)(I[F'A''] - I^2[F'A^{(3)}])$
ϵ_1	$4(D + 1)I^3[F'A^{(3)}]$
δ	$I^4[FA^{(4)}]$

Appendix B: Tables for the coefficient functions of external operators

TABLE VI: For $F(y)\partial_0^2\partial_0'^2A(y)$.

External operators	The functions they act on
α	$I^4[FA^{(4)}]$
β_1	$\frac{1}{2}(D-2)^2I^4[FA^{(4)}]$
β_2	$-(D-2)(D-1)I^4[FA^{(4)}]$
γ_1	$\frac{1}{2}(D-2)^3(D-1)I^4[FA^{(4)}] - 2(D-1)(I^2[F'A'] + I^3[F'A''] + I^4[F'A^{(3)}])$ $+ (12 + 40y - 26y^2)(I[F'A''] + I^2[F'A^{(3)}]) + 8(D-1)(I^2[F'A''] + I^3[F'A^{(3)}]) + (-48y + 64y^2 - 20y^3)I[F'A^{(3)}]$ $+ (D-1)(-24I^2[F'A^{(3)}] + 28I[yI[F'A^{(3)}]] - 14I[y^2I[F'A^{(3)}]]) - 20(D-1)I[yI[F'A''] + yI^2[FA^{(3)}]]$
γ_2	$\frac{1}{2}(D-2)^2yI^3[FA^{(4)}] - 32y(I[F'A''] + I^2[FA^{(3)}]) - 12(D-1)(I^2[F'A''] + I^3[F'A^{(3)}])$ $+ (D-1)(24I^2[F'A^{(3)}] - 12I[yI[F'A^{(3)}]]) + (48y - 26y^2)I[F'A^{(3)}]$
γ_3	$\frac{1}{2}(D-2)^2(y^2I^2[FA^{(4)}] + yI^3[FA^{(4)}]) + \frac{1}{2}(D-2)^3yI^3[FA^{(4)}]$
δ	$-I^4[FA^{(4)}](y)$
ϵ_1	$\frac{1}{2}(D-2)2DI^4[FA^{(4)}] + 2(D-2)I[yI^2[FA^{(4)}]] - \frac{1}{2}(D-2)I[y^2I^2[FA^{(4)}]] - \frac{1}{2}(D-2)DI[yI^3[FA^{(4)}]]$ $-\frac{1}{4}(D-2)^3I^5[FA^{(4)}] - (D-2)^2I^4[FA^{(4)}] - (D-2)(D-1)I^4[FA^{(4)}] + (I^3[F'A'] + I^4[F'A''] + I^5[F'A^{(3)}])$ $+ 10I^2[yI[F'A''] + yI^2[F'A^{(3)}]] - 4(I^3[F'A''] + I^4[F'A^{(3)}]) + 7I^2[y^2I[F'A^{(3)}]] - 14I^2[yI[F'A^{(3)}]] + 12I^3[F'A^{(3)}]$
ϵ_2	$6(I^3[F'A''] + I^4[F'A^{(3)}]) + 6I^2[yI[F'A^{(3)}]] - 12I^3[F'A^{(3)}]$
ϵ_3	$-(D-2)yI^3[FA^{(4)}]$
ζ	$I^4[FA^{(4)}] + \frac{1}{2}(D-2)I^5[FA^{(4)}]$

TABLE VII: For $F(y)\nabla^2(\partial_0^2 + \partial_0'^2)A(y)$.

External operators	The functions they act on
γ_1	$8(D-1)^2(I^2[F'A''] + I^3[F'A^{(3)}]) - 8(D-1)(I[F'A''] + I^2[F'A^{(3)}]) + 16(D-1)y(I[F'A''] + I^2[F'A^{(3)}])$ $+ (-16(D-1)y + 8(D-1)y^2)I[F'A^{(3)}] - 8(D-1)(D+4)I^2[F'A^{(3)}]$ $+ 4(D-1)^2I[yI[F'A^{(3)}]]$
γ_2	$4(D-1)(I[F'A''] + I^2[F'A^{(3)}]) + I^3[F'A^{(3)}] - 12(D-1)(I^2[F'A''] + I^3[F'A^{(3)}])$ $+ 8(D-1)y(I[F'A''] + I^2[F'A^{(3)}]) + 2(D-1)y^2I[F'A^{(3)}] + 24(D-1)I^2[F'A^{(3)}] - 12(D-1)I[yI[F'A^{(3)}]]$
δ	$-I^4[FA^{(4)}]$
ϵ_1	$-(D-2)(D-1)I^4[FA^{(4)}] + 4(D-1)(I^3[F'A''] + I^4[F'A^{(3)}]) + 4(D+4)I^3[F'A^{(3)}] - 2(D-1)I^2[yI[F'A^{(3)}]]$
ϵ_2	$6(I^3[F'A''] + I^4[F'A^{(3)}]) + 6I^2[yI[F'A^{(3)}]] - 12I^3[F'A^{(3)}]$
ϵ_3	$-(D-2)yI^3[FA^{(4)}]$
ζ	$2I^4[FA^{(4)}] + \frac{1}{2}(D-2)I^5[FA^{(4)}]$

TABLE VIII: For $F(y)\nabla^2(\partial_0\partial_0'^2)B(y)$.

External operators	The functions they act on
γ_1	$2(D-1)(I[F'B'] + I^2[F'B''] + I^3[F'B^{(3)}]) - 4(D^2-1)(I^2[F'B''] + I^3[F'B^{(3)}])$ $+ ((4-2y)(D-1))(I[F'B''] + I^2[F'B^{(3)}]) + (8(D-1)y - 2(D-1)y^2)I[F'B^{(3)}]$ $+ 4((D-1)(D+4))I^2[F'B^{(3)}] - 2((D-1)(D+4))I[yI[F'B^{(3)}]]$
γ_2	$-12(D-1)I^2[F'B^{(3)}]$
δ	$\frac{1}{2}I^4[FB^{(4)}]$
ϵ_1	$\frac{1}{2}(D-2)(D-1)I^4[FB^{(4)}] + 2(D+1)(I^3[F'B''] + I^4[F'B^{(3)}]) + (D+4)I^2[yI[F'B^{(3)}]] - 2(D+4)I^3[F'B^{(3)}]$
ϵ_2	$\frac{1}{2}yI^3[FB^{(4)}] + 6I^3[F'B^{(3)}]$
ϵ_3	$\frac{1}{2}y^2I^2[FB^{(4)}] + \frac{1}{2}(D-1)yI^3[FB^{(4)}]$
ζ	$-I^4[FB^{(4)}] - \frac{1}{4}(D-2)I^5[FB^{(4)}]$

TABLE IX: For $\alpha \equiv (aa')^2 \square^2$ type terms.

$f_\alpha(y) = C_{1a} f_{\alpha(1a)}(y) + C_{1c} f_{\alpha(1c)}(y),$	
$f_{\alpha(1a)}(y)$	$I^4[FA^{(4)}]$
$f_{\alpha(1c)}(y)$	$I^4[FC^{(4)}]$
Total for $f_\alpha(y)$	
$-\frac{(D+2)(D+4)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{128(D-2)(D+1)}\left(\frac{1}{y}\right)^{D-2} - \frac{3H^4}{32\pi^4}\frac{1}{y}.$	

TABLE X: For $\beta_1 \equiv (aa')^2 H^2 \square$ type terms.

$f_{\beta_1}(y) = C_{1a} f_{\beta_1(1a)}(y) + C_{1c} f_{\beta_1(1c)}(y),$	
$f_{\beta_1(1a)}(y)$	$-(D-2)(D-1) I^4[FA^{(4)}]$
$f_{\beta_1(1c)}(y)$	$-(D-2)(D-1) I^4[FC^{(4)}]$
Total for $f_{\beta_1}(y)$	
$\frac{(D-1)(D+2)(D+4)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{128(D+1)}\left(\frac{1}{y}\right)^{D-2} + \frac{9H^4}{16\pi^4}\frac{1}{y}.$	

TABLE XI: For $\beta_2 \equiv aa'(a^2 + a'^2)H^2 \square$ type terms.

$f_{\beta_2}(y) = C_{1a} f_{\beta_2(1A)}(y) + C_{1c} f_{\beta_2(1C)}(y),$	
$f_{\beta_2(1a)}(y)$	$\frac{1}{2}(D-2)^2 I^4[FA^{(4)}]$
$f_{\beta_2(1c)}(y)$	$\frac{1}{2}(D-2)^2 I^4[FC^{(4)}]$
Total for $f_{\beta_2}(y)$	
$-\frac{(D-2)(D+2)(D+4)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{256(D+1)}\left(\frac{1}{y}\right)^{D-2} - \frac{3H^4}{16\pi^4}\frac{1}{y}.$	

TABLE XII: For $\gamma_1 \equiv (aa')^2 H^4$ type terms.

$f_{\gamma_1}(y) = C_{1a} f_{\gamma_1(1a)}(y) + C_{2a} f_{\gamma_1(2a)}(y) + C_{3a} f_{\gamma_1(3a)}(y) + C_b f_{\gamma_1(b)}(y) + C_{1c} f_{\gamma_1(1c)}(y) + C_{2c} f_{\gamma_1(2c)}(y) + C_{3c} f_{\gamma_1(3c)}(y),$	
$f_{\gamma_1(1a)}(y)$	$\frac{1}{2}(D-2)^3(D-1)I^4[FA^{(4)}] - 2(D-1)(I^2[F'A'] + I^3[F'A''] + I^4[F'A^{(3)}])$ $+ (12 + 40y - 26y^2)(I[F'A''] + I^2[F'A^{(3)}]) + 8(D-1)(I^2[F'A''] + I^3[F'A^{(3)}]) + (-48y + 64y^2 - 20y^3)I[F'A^{(3)}]$ $+ (D-1)(-24I^2[F'A^{(3)}] + 28I[yI[F'A^{(3)}]] - 14I[y^2I[F'A^{(3)}]]) - 20(D-1)I[yI[F'A'']] + yI^2[FA^{(3)}]$
$f_{\gamma_1(2a)}(y)$	$4(D^2-1)(I[F'A''] - I^2[F'A^{(3)}])$
$f_{\gamma_1(3a)}(y)$	$8(D-1)^2(I^2[F'A''] + I^3[F'A^{(3)}]) - 8(D-1)(I[F'A''] + I^2[F'A^{(3)}]) + 16(D-1)y(I[F'A''] + I^2[F'A^{(3)}])$ $+ (-16(D-1)y + 8(D-1)y^2)I[F'A^{(3)}] - 8(D-1)(D+4)I^2[F'A^{(3)}]$ $+ 4(D-1)^2I[yI[F'A^{(3)}]]$
$f_{\gamma_1(b)}(y)$	$2(D-1)(I[F'B'] + I^2[F'B''] + I^3[F'B^{(3)}]) - 4(D^2-1)(I^2[F'B''] + I^3[F'B^{(3)}])$ $+ ((4-2y)(D-1))(I[F'B''] + I^2[F'B^{(3)}]) + (8(D-1)y - 2(D-1)y^2)I[F'B^{(3)}]$ $+ 4((D-1)(D+4))I^2[F'B^{(3)}] - 2((D-1)(D+4))I[yI[F'B^{(3)}]]$
$f_{\gamma_1(1c)}(y)$	$\frac{1}{2}(D-2)^3(D-1)I^4[FC^{(4)}] - 2(D-1)(I^2[F'C'] + I^3[F'C''] + I^4[F'C^{(3)}])$ $+ (12 + 40y - 26y^2)(I[F'C''] + I^2[F'C^{(3)}]) + 8(D-1)(I^2[F'C''] + I^3[F'C^{(3)}]) + (-48y + 64y^2 - 20y^3)I[F'C^{(3)}]$ $+ (D-1)(-24I^2[F'C^{(3)}] + 28I[yI[F'C^{(3)}]] - 14I[y^2I[F'C^{(3)}]]) - 20(D-1)I[yI[F'C'']] + yI^2[FC^{(3)}]$
$f_{\gamma_1(2c)}(y)$	$4(D^2-1)(I[F'C''] - I^2[F'C^{(3)}])$
$f_{\gamma_1(3c)}(y)$	$8(D-1)^2(I^2[F'C''] + I^3[F'C^{(3)}]) - 8(D-1)(I[F'C''] + I^2[F'C^{(3)}]) + 16(D-1)y(I[F'C''] + I^2[F'C^{(3)}])$ $+ (-16(D-1)y + 8(D-1)y^2)I[F'C^{(3)}] - 8(D-1)(D+4)I^2[F'C^{(3)}]$ $+ 4(D-1)^2I[yI[F'C^{(3)}]]$
Total for $f_{\gamma_1}(y)$	
$\frac{(D-20)(D-2)(D-1)D(D+2)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{32(D+1)} \left(\frac{1}{y}\right)^D + \frac{(D-2)(D^5-35D^4+72D^3-100D^2-320D+256)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{256(D+1)} \left(\frac{1}{y}\right)^{D-1}$ $+ \frac{(D^7-51D^6+240D^5+532D^4-5008D^3+7072D^2+1472D-5632)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2})^2}{1024(D-2)^2(D+1)} \left(\frac{1}{y}\right)^{D-2} - \frac{27H^4}{16\pi^4} \frac{1}{y}.$	

TABLE XIII: For $\gamma_2 \equiv aa'(a^2 + a'^2)H^4$ type terms.

$f_{\gamma_2}(y) = C_{1a} f_{\gamma_2(1a)}(y) + C_{3a} f_{\gamma_2(3a)}(y) + C_b f_{\gamma_2(b)}(y) + C_{1c} f_{\gamma_2(1c)}(y) + C_{3c} f_{\gamma_2(3c)}(y),$	
$f_{\gamma_2(1a)}(y)$	$\frac{1}{2}(D-2)^2 y I^3[FA^{(4)}] - 32y(I[F'A''] + I^2[FA^{(3)}]) - 12(D-1)(I^2[F'A''] + I^3[F'A^{(3)}])$ $+ (D-1)(24I^2[F'A^{(3)}] - 12I[yI[F'A^{(3)}]]) + (48y - 26y^2)I[F'A^{(3)}]$
$f_{\gamma_2(3a)}(y)$	$4(D-1)(I[F'A'] + I^2[F'A''] + I^3[F'A^{(3)}]) - 12(D-1)(I^2[F'A''] + I^3[F'A^{(3)}])$ $+ 8(D-1)y(I[F'A''] + I^2[F'A^{(3)}]) + 2(D-1)y^2 I[F'A^{(3)}] + 24(D-1)I^2[F'A^{(3)}] - 12(D-1)I[yI[F'A^{(3)}]])$
$f_{\gamma_2(b)}(y)$	$-12(D-1)I^2[F'B^{(3)}]$
$f_{\gamma_2(1c)}(y)$	$\frac{1}{2}(D-2)^2 y I^3[FB^{(4)}] - 32y(I[F'C''] + I^2[FC^{(3)}]) - 12(D-1)(I^2[F'C''] + I^3[F'C^{(3)}])$ $+ (D-1)(24I^2[F'C^{(3)}] - 12I[yI[F'C^{(3)}]]) + (48y - 26y^2)I[F'C^{(3)}]$
$f_{\gamma_2(3c)}(y)$	$4(D-1)(I[F'C'] + I^2[F'C''] + I^3[F'C^{(3)}]) - 12(D-1)(I^2[F'C''] + I^3[F'C^{(3)}])$ $+ 8(D-1)y(I[F'C''] + I^2[F'C^{(3)}]) + 2(D-1)y^2 I[F'C^{(3)}] + 24(D-1)I^2[F'C^{(3)}] - 12(D-1)I[yI[F'C^{(3)}]])$
Total for $f_{\gamma_2}(y)$	
$\frac{3(D-2)D(D^2+D-2)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{8(D+1)}\left(\frac{1}{y}\right)^D + \frac{(D-2)(D^4+18D^3-17D^2+8D+8)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{64(D+1)}\left(\frac{1}{y}\right)^{D-1}$ $- \frac{(D^7-55D^6+536D^5-2424D^4+4916D^3-1840D^2-5552D+4544)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{1024(D-2)(D+1)}\left(\frac{1}{y}\right)^{D-2} + \frac{3H^4}{16\pi^4}\frac{1}{y}.$	

TABLE XIV: For $\gamma_3 \equiv aa'(a+a)^2H^4$ type terms.

$f_{\gamma_3}(y) = C_{1a} f_{\gamma_3(1a)}(y) + C_{1c} f_{\gamma_3(1c)}(y),$	
$f_{\gamma_3(1a)}(y)$	$\frac{1}{2}(D-2)^2(y^2 I^2[FA^{(4)}] + y I^3[FA^{(4)}]) + \frac{1}{2}(D-2)^3 y I^3[FA^{(4)}]$
$f_{\gamma_3(1c)}(y)$	$\frac{1}{2}(D-2)^2(y^2 I^2[FC^{(4)}] + y I^3[FC^{(4)}]) + \frac{1}{2}(D-2)^3 y I^3[FC^{(4)}]$
Total for $f_{\gamma_3}(y)$	
$\frac{3H^4}{16\pi^4}\frac{1}{y}.$	

TABLE XV: For $\delta \equiv (a^2 + a'^2)H^2\nabla^2\Box$ type terms.

$f_\delta(y) = C_{1a} f_{\delta(1a)}(y) + C_{3a} f_{\delta(3a)}(y) + C_b f_{\delta(b)}(y) + C_{1c} f_{\delta(1c)}(y) + C_{3c} f_{\delta(3c)}(y),$	
$f_{\delta(1a)}(y)$	$-I^4[FA^{(4)}]$
$f_{\delta(3a)}(y)$	$-I^4[FA^{(4)}]$
$f_{\delta(b)}(y)$	$\frac{1}{2}I^4[FB^{(4)}]$
$f_{\delta(1c)}(y)$	$-I^4[FC^{(4)}]$
$f_{\delta(3c)}(y)$	$-I^4[FC^{(4)}]$
Total for $f_\delta(y)$	
$\frac{H^4}{32\pi^4} \frac{1}{y}.$	

TABLE XVI: For $\epsilon_1 \equiv aa'H^2\nabla^2$ type terms.

$f_{\epsilon_1}(y) = C_{1a} f_{\epsilon_1(1a)}(y) + C_{2a} f_{\epsilon_1(2a)}(y) + C_{3a} f_{\epsilon_1(3a)}(y) + C_b f_{\epsilon_1(b)}(y) + C_{1c} f_{\epsilon_1(1c)}(y) + C_{2c} f_{\epsilon_1(2c)}(y) + C_{3c} f_{\epsilon_1(3c)}(y),$	
$f_{\epsilon_1(1a)}(y)$	$\frac{1}{2}(D-2)2DI^4[FA^{(4)}] + 2(D-2)I[yI^2[FA^{(4)}]] - \frac{1}{2}(D-2)I[y^2I^2[FA^{(4)}]] - \frac{1}{2}(D-2)DI[yI^3[FA^{(4)}]]$ $-\frac{1}{4}(D-2)^3I^5[FA^{(4)}] - (D-2)^2I^4[FA^{(4)}] - (D-2)(D-1)I^4[FA^{(4)}] + (I^3[F'A'] + I^4[F'A''] + I^5[F'A^{(3)}])$ $+10I^2[yI[F'A'']] + yI^2[F'A^{(3)}] - 4(I^3[F'A''] + I^4[F'A^{(3)}]) + 7I^2[y^2I[F'A^{(3)}]] - 14I^2[yI[F'A^{(3)}]] + 12I^3[F'A^{(3)}]$
$f_{\epsilon_1(2a)}(y)$	$4(D+1)I^3[F'A^{(3)}]$
$f_{\epsilon_1(3a)}(y)$	$-(D-2)(D-1)I^4[FA^{(4)}] + 4(D-1)(I^3[F'A''] + I^4[F'A^{(3)}]) + 4(D+4)I^3[F'A^{(3)}] - 2(D-1)I^2[yI[F'A^{(3)}]]$
$f_{\epsilon_1(b)}(y)$	$\frac{1}{2}(D-2)(D-1)I^4[FB^{(4)}] + 2(D+1)(I^3[F'B''] + I^4[F'B^{(3)}]) + (D+4)I^2[yI[F'B^{(3)}]] - 2(D+4)I^3[F'B^{(3)}]$
$f_{\epsilon_1(1c)}(y)$	$\frac{1}{2}(D-2)2DI^4[FC^{(4)}] + 2(D-2)I[yI^2[FC^{(4)}]] - \frac{1}{2}(D-2)I[y^2I^2[FC^{(4)}]] - \frac{1}{2}(D-2)DI[yI^3[FC^{(4)}]]$ $-\frac{1}{4}(D-2)^3I^5[FC^{(4)}] - (D-2)^2I^4[FC^{(4)}] - (D-2)(D-1)I^4[FC^{(4)}] + (I^3[F'C'] + I^4[F'C''] + I^5[F'C^{(3)}])$ $+10I^2[yI[F'C'']] + yI^2[F'C^{(3)}] - 4(I^3[F'C''] + I^4[F'C^{(3)}]) + 7I^2[y^2I[F'C^{(3)}]] - 14I^2[yI[F'C^{(3)}]] + 12I^3[F'C^{(3)}]$
$f_{\epsilon_1(2c)}(y)$	$4(D+1)I^3[F'C^{(3)}]$
$f_{\epsilon_1(3c)}(y)$	$-(D-2)(D-1)I^4[FC^{(4)}] + 4(D-1)(I^3[F'C''] + I^4[F'C^{(3)}]) + 4(D+4)I^3[F'C^{(3)}] - 2(D-1)I^2[yI[F'C^{(3)}]]$
Total for $f_{\epsilon_1}(y)$	
$\frac{(D^4+12D^3-24D^2-20D+16)H^2D^{-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{64(D^2-1)} \left(\frac{1}{y}\right)^{D-2} + \frac{27H^4}{32\pi^4} \frac{1}{y}.$	

TABLE XVII: For $\epsilon_2 \equiv (a^2 + a'^2)H^2\nabla^2$ type terms.

$f_{\epsilon_2}(y) = C_{1a} f_{\epsilon_2(1a)}(y) + C_{3a} f_{\epsilon_2(3a)}(y) + C_b f_{\epsilon_2(b)}(y) + C_{1c} f_{\epsilon_2(1c)}(y) + C_{3c} f_{\epsilon_2(3c)}(y),$	
$f_{\epsilon_2(1a)}(y)$	$6(I^3[F'A''] + I^4[F'A^{(3)}]) + 6I^2[yI[F'A^{(3)}]] - 12I^3[F'A^{(3)}]$
$f_{\epsilon_2(3a)}(y)$	$6(I^3[F'A''] + I^4[F'A^{(3)}]) + 6I^2[yI[F'A^{(3)}]] - 12I^3[F'A^{(3)}]$
$f_{\epsilon_2(b)}(y)$	$\frac{1}{2}yI^3[FB^{(4)}] + 6I^3[F'B^{(3)}]$
$f_{\epsilon_2(1c)}(y)$	$6(I^3[F'C''] + I^4[F'C^{(3)}]) + 6I^2[yI[F'C^{(3)}]] - 12I^3[F'C^{(3)}]$
$f_{\epsilon_2(3c)}(y)$	$6(I^3[F'C''] + I^4[F'C^{(3)}]) + 6I^2[yI[F'C^{(3)}]] - 12I^3[F'C^{(3)}]$
Total for $f_{\epsilon_2}(y)$	
$-\frac{(D-2)(13D^2+6D-16)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2}-1)^2}{128(D^2-1)}\left(\frac{1}{y}\right)^{D-2}.$	

TABLE XVIII: For $\epsilon_3 \equiv (a + a')^2H^2\nabla^2$ type terms.

$f_{\epsilon_3}(y) = C_{1a} f_{\epsilon_3(1a)}(y) + C_{3a} f_{\epsilon_3(3a)}(y) + C_b f_{\epsilon_3(b)}(y) + C_{1c} f_{\epsilon_3(1c)}(y) + C_{3c} f_{\epsilon_3(3c)}(y),$	
$f_{\epsilon_3(1a)}(y)$	$-(D-2)yI^3[FA^{(4)}]$
$f_{\epsilon_3(3a)}(y)$	$-(D-2)yI^3[FA^{(4)}]$
$f_{\epsilon_3(b)}(y)$	$\frac{1}{2}y^2I^2[FB^{(4)}] + \frac{1}{2}(D-1)yI^3[FB^{(4)}]$
$f_{\epsilon_3(1c)}(y)$	$-(D-2)yI^3[FC^{(4)}]$
$f_{\epsilon_3(3c)}(y)$	$-(D-2)yI^3[FC^{(4)}]$
Total for $f_{\epsilon_3}(y)$	
$\frac{(D+2)(D+4)H^{2D-4}\pi^{-D}\Gamma(\frac{D}{2})^2}{32(D^2-1)}\left(\frac{1}{y}\right)^{D-2} - \frac{H^4}{16\pi^4}\frac{1}{y}.$	

TABLE XIX: For $\zeta \equiv \nabla^4$ type terms.

$f_\zeta(y) = C_{1a} f_{\zeta(1a)}(y) + C_{2a} f_{\zeta(2a)}(y) + C_{3a} f_{\zeta(3a)}(y) + C_b f_{\zeta(b)}(y) + C_{1c} f_{\zeta(1c)}(y) + C_{2c} f_{\zeta(2c)}(y) + C_{3c} f_{\zeta(3c)}(y),$	
$f_{\zeta(1a)}(y)$	$I^4[FA^{(4)}] + \frac{1}{2}(D-2)I^5[FA^{(4)}]$
$f_{\zeta(2a)}(y)$	$I^4[FA^{(4)}]$
$f_{\zeta(3a)}(y)$	$2I^4[FA^{(4)}] + \frac{1}{2}(D-2)I^5[FA^{(4)}]$
$f_{\zeta(b)}(y)$	$-I^4[FB^{(4)}] - \frac{1}{4}(D-2)I^5[FB^{(4)}]$
$f_{\zeta(1c)}(y)$	$I^4[FC^{(4)}] + \frac{1}{2}(D-2)I^5[FC^{(4)}]$
$f_{\zeta(2c)}(y)$	$I^4[FC^{(4)}]$
$f_{\zeta(3c)}(y)$	$2I^4[FC^{(4)}] + \frac{1}{2}(D-2)I^5[FC^{(4)}]$
Total for $f_\zeta(y)$	
0.	